Conventional and Non-Conventional Monetary Policy: Between Core and Periphery

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Abstract

This paper explores the effectiveness of government bond and corporate security purchases by a central bank within a calibrated two-country New-Keynesian model featuring a banking sector (an extension of Gertler and Karadi (2011) and Andrade et al. (2016)) and a two-country monetary union. Focusing on the Eurozone and motivated by the extended asset purchase programme conducted by the ECB we calibrate key parameters to match Core (Germany, France, Netherlands) and Periphery (Portugal, Italy, Ireland, Greece, Spain) data. We find that corporate security purchases have a stronger impact on inflation and on lift-off time from the Effective Lower Bound than equivalent government bond purchases. Corporate securities are claims on capital used in the firm's production function. This finding is in line with the ones of Gertler and Karadi (2013) for the U.S. economy.

Keywords: Monetary Policy, Asset Purchases, Monetary Union

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1 Introduction

The Great recession disrupted the traditional transmission mechanism of monetary policy and led the major Central Banks into the unchartered territory of non conventional measures. After 2008, the European Central Bank used non standard measures at first in conjunction with standard monetary policy, to allow its correct operation amid market disruption. Since 2015, as the interest rate reached its Effective Lower Bound (ELB), the APP (Asset Purchase Programme) complemented the conventional monetary policy. Between 9 March 2015 and 19 December 2018 the Eurosystem conducted net purchases of public sector securities under the public sector purchase programme (PSPP). Additionally, as of 2016 the ECB added to the APP the net purchases of corporate sector bonds under the corporate sector purchase programme (CSPP)[1].

The use of non conventional measures and their effectiveness in times of distress and even as a substitute for interest-rate based monetary policy has been widely debated. A related question concerns the relative effect of the different parts of the programme. Finally, as the sovereign and banking debt crisis particularly hit periphery countries or the euro area, the analysis of the possible asymmetric effects of common policies became important.

In this paper, we explore the effectiveness of government bond and corporate security purchases within a calibrated two-country New-Keynesian model with a banking sector and a monetary union. We combine three important dimensions of the European experience after the crisis and analyze their interactions.

i) We focus on the effect of unconventional policies in three different regimes: during normal times, in "difficult times" when the banking system lacks liquidity and the transmission of a change in the interest rates is impaired, and under the ELB,

[1] A smaller part of the APP also includes the asset-backed securities purchase programme (ABSPP) and the covered bond purchase programme (CBPP)

when rates simply cannot be reduced further and unconventional policies essentially act as a substitute.

ii) We deal with the possible asymmetric effects of common policies by examining them in the context of a two-country monetary union, where the two countries represent the core of the euro area and the countries that most suffered during the recession (periphery), respectively. Due to the rich structure of our banking sector, where Core and Periphery banks are able to hold Core and Periphery government bonds and corporate securities, we are able to analyze the transmission of shocks through the interbank market. The two-country setup further allows considering different economic structures and banking systems and to assess their role in the transmission of policy shocks.

iii) Finally, we distinguish between two different classes of assets targeted by APP. More specifically, we differentiate between long-term government bonds and stocks, thereby reproducing the different mechanism behind the PSPP and CSPP. The presence of short-term investment and long-term government bonds also accounts for different maturities.

Our initial findings pertain to the propagation of shocks within our Monetary Union. We find that a capital destruction shock in the Periphery causes a fall in the output of the entire union, and this propagation is amplified if financial markets are fluid such that banks and households can freely trade government bonds. To have a strong real effect on the monetary union we need to assume extreme fluidity of the financial markets. However, even under more realistic calibration we are still able to find propagation effects to the financial market from a 1% capital destruction shock in the Periphery.

Our second focus of the paper is to analyze the interaction between the ELB and the effectiveness of various QE programmes implemented by the ECB. We calibrate the model to 2012 and use a capital destruction shock in each region followed by a demand shock, thereby forcing the economy to the Effective Lower Bound. We are able to see that, under our calibration and shocks, the real GDP loss due to the inability of the central bank to lower the interest rate is roughly 1% at its peak during 6 quarters at the ELB.

The loss of the instrument of the policy rate at the ELB forced the ECB to turn to unconventional monetary policy. We find that corporate security purchases are more effective than government bond purchases. This finding is mechanical due to our collateral constraint a la Gertler and Karadi (2013). However, due to calibrating to the eurozone, and hence an economy with less reliance on corporate securities, we find that the effectiveness of corporate security purchases versus government bond purchases are dampened.

2 The European Experience

In their survey of 20 years of ECB experience, Hartmann and Smets (2018) identify four time periods, and implicitly three regimes. In the first regime (from 1999 to 2007) the ECB was able to use standard monetary policy to achieve its inflation objective. The policy rate during this period varied between 2 and 5 percent, far from the ELB,and the ECB operated accordingly to the so-called Separation Principle: liquidity operations and asset purchases addressed malfunctioning interbank money markets and sovereign bond markets and thereby facilitated the transmission of monetary policy, interest rates focused on maintaining price stability over the medium term.

A second regime (from 2008 to 2013) started with the collapse of Lehman Brothers, the following Great Recession and the beginning of the sovereign crisis. In these “difficult times” the ECB maintained the Separation Principle, but conventional and non conventional policies were used jointly. The ECB lowered its key policy rate to an unprecedented level of 1%. At the same time, to respond to the increased demand for liquidity and reduce the risk of financial disruptions, the ECB introduced a number of non conventional measures. Starting in October 2009, the Main Refinancing Operations (MROs) were conducted with full allotment, in practice letting demand decide the amounts allocated at the MRO interest rate. Additional measures included the expansion of the list of marketable assets accepted as collateral in Eurosystem credit operations, and a Covered Bond Purchase Programme.

In the third and final regime (from 2014 onwards) the ECB used non conventional measures to overcome the Effective Lower Bound on interest rates. When the Effective Lower Bound (ELB) of the interest rate was approached, non conventional policies acted as a substitute while the ECB counteracted the risk of deflation and attempted at bringing inflation back to close to two percent. During this phase, policies such as funding for lending, forward guidance and (most of all) quantitative easing determined an expansion of the balance sheet of the ECB, both in size and variety of assets. The ECB's assets reached 3 trillions in the course of 2019; most of these securities are held for monetary policy purposes (see Figure 1).

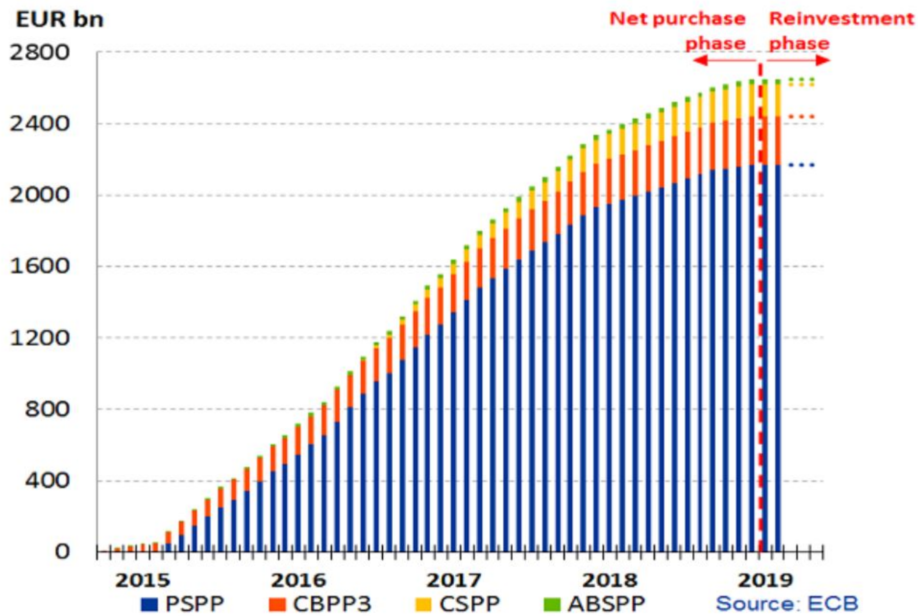


Fig. 1: Operations conducted by the Eurosystem in the context of implementing its monetary policy. Source: ECB.

3 Literature Review

From a theoretical perspective one of the most influential works on Quantitative Easing is from Eggertsson and Woodford (2003a, 2004), where the authors analyse the effects of open-market operations. Their main finding is that

> Quantitative easing that implies no change in the interest-rate policy should neither stimulate real activity nor halt deflation; and this is equally true regardless of the kind of assets purchased by the central bank. (Eggertsson and Woodford, 2004)

The previous quote was also theorized earlier in Wallace (1981).

Theoretically this view has been challenged using models that include the short-term interest rate at the zero lower bound. An example of this is from Bernanke and Reinhart (2004) who present a model with the ZLB and financial frictions, which during crises prevent arbitrage across asset classes and drive changes in term premia of assets. As a consequence,

> QE can take the risk of default out of the balance sheet of the banks and into the balance sheet of the central bank, reducing the extent of the credit crunch and increasing the effective supply of safe asset. (Reis, 2016)

If the distress in the economy is due to a fragile financial sector then credit easing, purchasing risky assets and providing safe reserves, reduces the risks and the fragility of financial intermediation.

We break the irrelevance result[2] of quantitative easing theoreticized by Wallace (1981) in the same spirit as Gertler and Karadi (2011). The main friction is a collateral constraint, or an incentive compatibility constraint, which means that bankers are

[2]Another technique to break the irrelevance result is through a preferred-habit model of the term strucuture of interest rates, whereby agents in the model prefer to hold assets of different maturities, this has been popularised by Vayanos and Vila (2009) and more recently features heavily in Ray (2019)

only trusted to hold a certain amount of corporate securities and government bonds and they take this into account when maximising their lifetime net worth. Since there are limits to arbitrage, central bank intermediation increases overall asset demand and does not solely displace the private intermediation one-for-one, this increased demand increases the price. The Gertler and Karadi (2011) paper has formed the basis of many further research works, such as their own theoretical study calibrated to the U.S. experience of using quantitative easing, as seen in Gertler and Karadi (2013). More recently, this work has been extended to also analyse the impact on the eurozone by the quantitive easing and forward guidance conducted by the European Central Bank, Andrade et al. (2016).

A salient feature of Gertler and Karadi (2013) is that corporate security purchases (defined as claims on firm's capital) have a larger impact on the economy than government bond purchases, due to their riskier nature as they are easy to be absconded with compared to government bonds and therefore intermediation by the central bank is more beneficial. Kurtzman and Zeke (2018) show that if central bank purchases from large firms this reduces the incentive to invest from smaller firms whose debt is not purchased and therefore induces non-neglible misallocation costs. If these misallocation costs are sizable then securities purchases can be less effective than government bond purchases in stimulating the economy. Since we do not embed a heterogeneous firm structure into our model this misallocation effect is not present and thus we also find that corporate securities purchases are more effective than government bond purchases.

A closely related paper that analyses quantitative easing within a two-country monetary union is Bletzinger et al. (2018). They include short-term and long-term government bonds in a symmetric and asymmetric monetary union whilst taking the fiscal structure of each country seriously. Our paper differs from theirs by focusing on the effectiveness of government bond purchases versus private security purchases, which is not included in their model. On the other hand, to keep the model tractable our fiscal structure is purposely kept simple. Another paper that focuses on a two-country DSGE model of a monetary union estimated to fit Core and Periphery of the eurozone is Poutineau and Vermandel (2015). Their work focuses on the cross-border transmission of shocks and find that national variables, for example regional production and consumption, are less sensitive to financial shocks whilst investment is more sensitive. Our findings are in a similar vein as we experiment with opening and closing the financial transmission in our model to study how shocks in our region propagate to the other region. Moreover, they find little difference in the sensitivity of national variables to shocks when they move from banking autarky to a cross-border banking parameterization. Although we do not completely turn off the banking sector, as in Poutineau and Vermandel (2015), the limited difference in movement in national variables is echoed in our work when we experiment with fluid and rigid banking sectors. The closest paper to ours is Auray et al. (2018), who evaluate PSPP with and without the ELB, but they do not focus on CSPP or maturity effect from long term government bonds and also differ by adding government default risk to their model. One way in which we try to demonstrate an added riskiness of the Periphery region is through a higher probability of Periphery banks failing, thus leading to a higher Periphery government bond premium compared to the Core region.

Although our paper does not focus on the empirical results from the European Central Bank's programmes, we utilise evidence by Andrade et al. (2016) to motivate our work. They find that "the programme produced significant effects upon announcement, on 22 January 2015" and that these effects are expected to last "approximately as long as in the case of standard monetary policy announcements." There also seems to be an effect other than the signalling channel, more specifically:

> We show that average yields (in basis points) plotted relative to the day prior to the PSPP announcement, dropped on average by about 13 basis

points after the announcement and an additional 14 basis points after the implementation. Andrade et al. (2016).

Further compelling evidence of the impact of QE on the economy is shown in Haldane et al. (2016), who focus on the experience of the main economies that conduced QE. They find reasonably strong evidence the suggest that QE has had an impact on financial markets, loosing credit constraints, as well as on the real economy through temporarily boosting GDP and prices.

4 Model - Two Countries

4.1 Layout

We build a two-country New Keynesian model à la Galí (2015) with a banking sector motivated by Gertler and Karadi (2011). There are two regions denoted as Core and Periphery, one central bank and two fiscal authorities. Figure 2 below represents the model layout, stars denote the Periphery region. Households are either workers or bankers. Workers supply labour, deposit into banks and hold Core and Periphery government bonds. Bankers wish to maximise their lifetime net worth taking into account their budget constraint, collateral constraint and the probability of survival (σ). Bankers hold Core and Periphery government bonds as well as corporate securities, which are modeled as claims on capital. The governments are kept purposely simple and solely finance the net interest on a fixed amount of government bonds through lump-sum taxes. The central bank sets the interest rate on safe deposits for both regions following a Taylor rule and conducts asset purchases dictated currently by an exogenous AR(1) shock[3].

[3]Although an AR(2) process more closely represents asset purchases and the expected path of these purchases by the ECB, we currently use an AR(1) for simplicity.

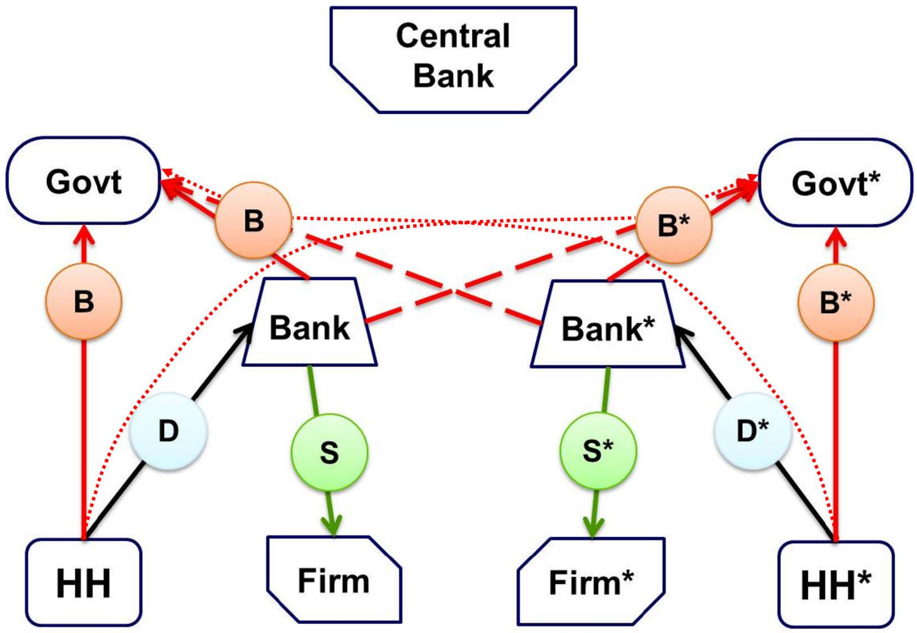


Fig. 2: Basic structure of the model. Stars represents Periphery. B denotes government bonds, D deposits, S corporate securities.

4.2 Households

Our model derivation is focused on the Core region since the theoretical setup between Core and Periphery economies are symmetric. Households in the Core region (symmetric for Periphery) gain utility from consumption and disutility from working. The utility function includes habit formation, as this is shown to improve the empirical fit of the model, and takes the form:

$$E_t \sum_{j=0}^{\infty} \beta_{t,t+j} \left[ln(C_{t+j} - hC_{t+j-1}) - \chi \frac{(L_{t+j})^{1+\varphi}}{1+\varphi} \right] \zeta_t \tag{4.1}$$

with $0 < \beta < 1$, $0 < h < 1$, $\chi > 0$ and $\varphi > 0$ all taking values calibrated to the euro area. ζ_t is added as a preference shifter (pure demand shock) and will be assumed to follow a persistent AR(1) process. Labour is a composite of heterogeneous labour services provided by the household and the economy is considered to be at limit where it becomes cashless as in Woodford (2011) and Galí (2015), hence the convenience yield of real money balances are ignored. There is a unit continuum of households within the model, where a household belongs to the Core region if $j = [0, n)$ and the Periphery region if $j \in [n, 1]$. Households that are part of the Core are able to purchase goods from the Periphery, and vice-versa, with Periphery goods being denoted with superscript * when clarification is necessary. Consumption by the Core households of Core goods is given by c and Core household consumption of Periphery goods is denoted as c^*. Aggregate consumption C_t in the Core is the share of consumption of Core goods c and Periphery goods c^*, which is a Dixit-Stiglitz aggregator of consumption goods from each region taking into account a home bias[4]:

$$C_t \equiv \left[(\nu)^{\frac{1}{\theta_T}} (c_t)^{\frac{\theta_T - 1}{\theta_T}} + (1 - \nu)^{\frac{1}{\theta_T}} (c_t^*)^{\frac{\theta_T - 1}{\theta_T}} \right]^{\frac{\theta_T}{\theta_T - 1}} \tag{4.2}$$

The above equation characterizing total consumption in the Core region allows for home bias through $\nu \in [0, 1]$. This home bias, therefore, affects the price index for the region and the currency union. θ_T measures the elasticity of substitution between goods in the Core and Periphery.

Price index for the Core region takes the form:

$$P_t \equiv \left[\nu (p_t)^{1-\theta_T} + (1 - \nu)(p_t^*)^{1-\theta_T} \right]^{\frac{1}{1-\theta_T}}$$

The household can consume either Core or Periphery final goods and deposit their savings into a bank in the Core region. The household receives a wage for working, the net worth of a Core bank Ξ_t when the bank ceases business, transfers from the government T_t, interest payments from her previous period deposits $R_t D_t$. It needs to be noted that the rate of return for deposits here are in real terms and therefore are deflated by the price index P_t. Households' save using short-term bank deposits

[4]For further details on the Dixit-Stiglitz aggregation see the Appendix Section A.

D_{ht} and long-term government bonds B_{ht}, where subscript h means that is held by the household, subscript b means that is from the banking sector. To account for limited participation in asset markets by households, which provides us with limited arbitrage among assets, holdings of government bonds comes with a cost equal to the percentage of total government bonds held above a threshold $\bar{B}_h$[5]. As is typically assumed the long-term government bonds are perpetuities, however, to add realism to our government bonds we take inspiration from Woodford (2001) and allow the bonds to decay at rate ρ. Following Auray and Eyquem (2017) the decay rate ρ is calibrated to match a 10 year or 40-quarter government bond with $\mathcal{M}$ denoting the maturity of the bond and β the households discount rate.

$$\mathcal{M} = \frac{1}{1-\beta\rho} = 40$$

Therefore the interest rate on the government bond can be defined as:

$$R_{b,t} = \frac{1+\rho Q_{b,t}}{Q_{b,t-1}}$$

The budget constraint can be shown to be:

$$\begin{aligned} C_t + D_t + Q_{b,t}[B_{h,t} + \frac{1}{2}\kappa(B_{h,t} - \bar{B}_h)^2] + Q^*_{b,t}[B^*_{h,t} + \frac{1}{2}\kappa^*(B^*_{h,t} - \bar{B}^*_h)^2] \\ = \frac{W_t}{P_t}L_t + \Xi_t + T_t + R_t D_{t-1} + R_{b,t}Q_{b,t-1}B_{h,t-1} + R^*_{b,t}Q^*_{b,t-1}B^*_{h,t-1} \end{aligned}$$

Households optimize equation (4.1) using $\{C_t, L_t, B_{ht}, B^*_{ht}, D_{ht}, W_t\}$ subject to the budget constraint. Given the assumption of flexible wages, the real wage will be a markup over the marginal rate of substitution. Collating the first order conditions of this problem:

[5]Adding in the long-term bonds allows for asset purchase analysis of government bonds vs private loans (PSPP vs CSPP), it is possible to simplify this further by taking out the role of long-term government bonds

$$E_t\left[\beta\frac{U_{C,t+1}}{U_{C,t}}R_{t+1}\right] = E_t[\Lambda_{t,t+1}R_{t+1}] = 1$$

$$\frac{W_t}{P_t} = \chi(L_t)^{\varphi}\zeta_t\frac{1}{U_{C,t}}$$

$$B_{h,t} = \bar{B}_h + \frac{E_t[\Lambda_{t,t+1}(R_{b,t+1} - R_{t+1})]}{\kappa}$$

$$B^*_{h,t} = \bar{B}^*_h + \frac{E_t[\Lambda_{t,t+1}(R^*_{b,t+1} - R_{t+1})]}{\kappa^*}$$

Where $\Lambda_{t,t+1} \equiv \beta\frac{U_{C,t+1}}{U_{C,t}}$ is utilised.

4.3 Banks

The banking system is modeled as a two-country extension of Gertler and Karadi (2013) or Andrade et al. (2016). Banks receive deposits from households and use these to make loans to firms and purchase government bonds.

Lending goes entirely to domestic non-financial firms. The return for holding a claim on a non-financial firm $R_{k,t+1}$ is equal to the marginal productivity of the capital lent to the firm (Z_{t+1}) plus the value of this capital leftover (after depreciation) $(1-\delta)Q_{t+1}$ divided by the cost of this asset today (Q_t, or the cost of capital). Region specific capital quality shocks are given as in Gertler and Karadi (2011) by ξ_{t+1}. This can be summarised as:

$$R_{k,t+1} = \frac{Z_{t+1} + (1-\delta)Q_{s,t+1}}{Q_{s,t}}\xi_{t+1}$$

Additionally, banks have access to a common financial market through their ability to purchase government bonds from the domestic and foreign government. Focusing on the Core region and writing the relative preference for domestic and foreign government bond holdings as a CES function, as in Auray et al. (2018), Core and Periphery government bonds are only partially substitutable, with ι being the elasticity of substitution. Additionally, a parameter υ is added to calibrate the well-documented home bias in government bonds.

$$b_{CES,t} = \left(\upsilon_b{}^{\frac{1}{\iota}} \, b_t{}^{\frac{\iota-1}{\iota}} + (1-\upsilon_b)^{\frac{1}{\iota}} \, b_t^*{}^{\frac{\iota-1}{\iota}} \right)^{\frac{\iota}{\iota-1}}$$

The interest rate received on the bank's government bond portfolio can be written in a similar fashion as the CES government bond structure and is defined as $R_{CES,t}$. The value of the government bond portfolio held by the Core bank is given by:

$$Q_{b,CES,t} b_{CESt} = Q_{b,t} b_t + Q^*_{b,t} b^*_t$$

The banks' activities in the balance sheet includes claims on firms $Q_{s,t}s_t$ (at a regional market price), domestic government bond holdings $Q_{b,t}b_t$ and foreign government bond holdings $Q^*_{b,t}b^*_t$. This is equal to the banks' net worth n_t plus deposits received this period d_t. Combining the banks interim balance sheet and flow of funds gives the evolution of the bank's net worth[6]:

$$n_t = (R_{k,t} - R_t) Q_{s,t-1} s_{t-1} + (R_{b,CES,t} - R_t) Q_{b,CES,t-1} b_{CES,t-1} + R_t n_{t-1}$$

We now turn to the maximization problem of the banker. As bankers are detached from the household, their objective is to maximise their net worth and the payments to the household. Their discount factor is the same as the households' intertemporal marginal rate of substitution $\Lambda_{t,t+j}$, augmented with the probability $1-\sigma$ that the banker will cease business and return to the household, transferring the remaining net worth to the household as a lump-sum payment. The maximization problem can be written as:

[6]For a full derivation of the banks problem see Appendix C

$$\max_{b_{CES_t}, s_t} V_t = E_t \sum_{j=0}^{\infty} (1-\sigma)\sigma^{j-1} \Lambda_{t,t+j} n_{t+j} \tag{4.3}$$

Finally, we add the incentive compatibility constraint. As in Gertler and Karadi (2011), the bankers are able to divert a proportion of funds back to their own household. The incentive to default reduces the amount the depositors are willing to lend to the banks. It is assumed here that diverting funds from private loans (loans made to firms) is easier than diverting funds from government bonds. Specifically, the banker can divert θ from their private loans and $\theta\Delta$, with $0 < \Delta < 1$, from government bonds. We assume that it is equally difficult to abscond with Core government bonds as it is with Periphery government bonds. The incentive compatibility constraint is then given as:

$$V_t \geq \theta Q_{s,t} s_t + \Delta\theta(Q_{b,CES,t} b_{CESt}) \tag{4.4}$$

Adding a moral hazard or costly enforcement problem is essential to make financial markets non-frictionless and therefore to induce non-neutral asset purchases by the central bank, breaking the irrelevance result of Eggertsson and Woodford (2003b).

The solution of the maximization problem under compatibility constraint results in a risk-adjusted leverage constraint, where ϕ_t is the leverage ratio and the below inequality will hold with equality.

$$Q_{s,t} s_t + \Delta(Q_{b,CES,t} b_{CESt}) \leq \phi_t n_t$$

The leverage ratio, ϕ_t, is an adjusted measure of assets to net worth representing the maximal value of assets the bank is able to hold without violating the incentive compatibility constraint. Tighter scrutiny on the bank reduces the ability of the bank to divert funds and increases trust in the bank, lowering θ and increasing the amount a bank can hold and the leverage ratio ϕ_t: [7]

$$\phi_t = \frac{E_t \Omega_{t,t+1} R_{t+1}}{\theta - E_t \Omega_{t,t+1}(R_{k,t+1} - R_{t+1})}$$

Additionally, a bank must be indifferent between investing in firms or purchasing government bonds. Therefore, in expectation, the following arbitrage condition must hold over Core and Periphery government bonds and loans to non-financial Home firms:

$$\Delta E_t \Omega_{t+1}(R_{k,t+1} - R_t) = E_t \Omega_{t+1}(R_{b,CES,t+1} - R_t) \tag{4.5}$$

4.4 Aggregation of banks

All banks within a given region are identical; [8] the equivalent equilibrium conditions are therefore given as the incentive compatibility constraint and the evolution of total net worth.

$$Q_{s,t} S_{b,t} + \Delta(Q_{b,CES,t} B_{b,CES,t}) \leq \phi_t N_t$$

$$N_t = \sigma \left((R_{k,t} - R_t) \frac{Q_{s,t-1} S_{b,t-1}}{N_{t-1}} + (R_{b,CES,t} - R_t) \frac{Q_{b,CES,t-1} B_{b,CES,t-1}}{N_{t-1}} + R_t \right) N_{t-1} + \omega$$

[7] Bankers' problem is is laid out in greater detail in Appendix C. The leverage ratio is found by guess-and-verify, where $\Omega_{t,t+1} = \Lambda_{t,t+1}[1 - \sigma + \sigma\theta\phi_{H,t+1}]$, is the banks' augmented discount factor, reflecting the shadow value of a unit of net worth.

[8] Uppercase variables are analogous to their lowercase counterparts. Therefore $S_{b,t}$ is defined as the aggregate claims by financial firms on non-financial firms within the economy. $B_{b,t}$ and $B^*_{b,t}$ are defined as total Core and Periphery, government bonds, respectively, held by Core banks.

4.5 Regional Governments

There are two identical regional governments. Government bonds are assumed to be in fixed supply and their quantities are calibrated to the debt-over-GDP of each region. The governments pay net interest on bonds and balance their budget through taxes levied on the households in their region.

$$(R_{b,t} - 1)\bar{B}_t = T_t$$

The total amount of government bonds of the Core region is:

$$B_t = \bar{B}_t$$

and the total amount of government bonds in the monetary union is exogenous and defined as:

$$B_t^U = \bar{B}_t + \bar{B}_t^*$$

4.6 Central Bank

A central bank conducts monetary policy for the whole union. The Central Bank's objective is to set the nominal interest rate in order to minimize deviations of inflation from its steady state (or target) value and output from its natural level (the level of output that would prevail if no frictions were applied to the model).

Conventional monetary policy sets the common interest rate on deposits following a non-linear interest rate rule defined on a harmonized index of consumer prices, P_t^U, and the growth (inflation) of these prices Π_{t+1}^U within the monetary union. We assume interest rate smoothing governed by the parameter ϕ_i.

$$1+i_t^U = \max\left\{\left[\frac{1}{1+i^U}\left(\frac{\Pi_t^U}{\Pi^U}\right)^{\phi_\Pi}\left(\frac{Y_t^U}{Y^U}\right)^{\phi_y}\right]^{1-\phi_i}\left[1+i_{t-1}^U\right]^{\phi_i}, 1\right\}$$

i_t is the net nominal interest rate. The nominal interest rate maps into the real interest rate on deposits through deflating by inflation:

$$1+i_t^U = R_t^U \Pi_{t+1}^U$$

The Harmonised Index on Consumer Prices (HICP) is given by weighting the price levels of both regions by their relative size, n.

$$P_t^U = \left(P_t\right)^n \left(P_t^*\right)^{1-n}$$

After hitting the Zero-Lower-Bound on interest rates the central bank can use Unconventional Monetary Policy (UMP) to stimulate the economy. UMP in this model takes the form of purchasing government bonds or claims on financial firms, thus increasing their price within the economy and lowering the excess return on these assets. For comparability purposes the central bank purchases of private assets , $\psi_{S;t}$, and government bonds , $\psi_{B,t}$, are expressed as a share of GDP. Following the ECB's practice of purchasing according to capital key[9], the share of bonds/assets purchased is assumed proportional to the size of the two countries and determined by where the bond/asset originated from and not the location of the bank that holds it. [10] Subscript g is used to denote assets held by the central bank.

[9]The capital key governs the proportion of bonds the ECB can buy from each country.

[10]As in Gertler and Karadi (2011), the central bank must pay an additional efficiency cost τ to hold onto these assets. In their model, this cost ensures that the central bank does not take over the intermediation role of a financial firm permanently. Assuming the central bank is less efficient can be rationalized through additional monitoring costs that a central bank will need to complete while holding the asset. As our asset purchases are stylized and do not follow an asset purchasing rule that depends on interest rate spreads, this efficiency cost is redundant and added in an attempt to more accurately portray the costs and benefits of asset purchases.

$$B_{g,t} = \varphi_{B,t} B_t$$

$$S_{g,t} = \varphi_{S,t} S_t$$

These purchases follow an AR(1) process

$$\varphi_{B,t} = \rho_B \varphi_{B,t-1} + \varepsilon_{B,t}$$

$$\varphi_{S,t} = \rho_S \varphi_{S,t-1} + \varepsilon_{S,t}$$

The central bank finances its purchases through issuing central bank reserves, $D_{g,t}$, which pay the safe interest rate R_{t+1}, and from interest on previously held government bonds and corporate securities.

This short term debt is issued to households. An equivalent, but more realistic way, to model central bank reserves is to have them held by banks. If private banks are unable to abscond with central bank reserves, which are held at the central bank, then this will lead to identical results. The balance sheet of the central bank is:

$$Q_{s,t} S_{g,t} + Q^*_{s,t} S^*_{g,t} + Q_{b,t} B_{g,t} + Q^*_{b,t} B^*_{g,t} = D_{g,t}$$

The total amount of the government bonds in the economy is exogenous fixed, calibrated to the debt-to-GDP ratio of each region. Central bank purchases of government bonds therefore push up the price of this asset and push down the bond yield, lowering the government bond premium.

$$B^U_t = B_t + B^*_t + B_{g,t} + B^*_{g,t}$$

Unlike government bonds, which are in a positive fixed supply, corporate securities (that are claims on capital) can increase due to rising investment within the economy. When the central bank purchases these assets they are taking over the intermediation of firms without the limit of the moral hazard problem faced by private banks. Therefore the total amount of securities in the Core region is given by those held by the private bank, $S_{b,t}$, and central bank, $S_{g,t}$.

$$S_t = S_{b,t} + S_{g,t}$$

4.7 Rest of the model

The rest of the model follows a two-country version of the standard New Keynesian setup à la Galí (2015) .

4.8 Non-Financial Firms: Intermediate good producers

The intermediate good firms produce their goods following a Cobb-Douglas production function using capital, K_t, and labour , L_t, available within the region. Intermediate goods produced in the Core region are sold at price $P_{m,t}$. Output in the Core region, Y_t, is produced using a technology common to all intermediate good producers A_t. The output elasticity of capital is given by α and labour elasticity is $1-\alpha$.

$$Y_t = A_t(\xi_t K_t)^{\alpha}(L_t)^{1-\alpha}$$

The firms demand for labour, where $P_{m,t}$ is the price of intermediate goods, is equal to the marginal productivity of labour.

$$\frac{W_t}{P_t} = P_{m,t}(1-\alpha)\frac{Y_t}{L_t}$$

Gross profit per unit of capital in the Core region is given by Z_t:

$$Z_t = P_{m,t}\alpha\frac{Y_t}{\xi_t K_t}$$

The capital stock evolves according to regional investment I_t, a region-specific capital quality shock ξ_{t+1}, and depreciates at rate δ:

$$K_{t+1} = I_t + (1-\delta)K_t\xi_{t+1}$$

4.9 Capital good producers

Capital goods producers are owned by households and therefore discount their expected future profits at the stochastic discount rate $\Lambda_{t,t}$. They produce capital through investment I_t, using final output as an input.[11] They sell capital to firms at the price Q_t. Therefore they choose I_t to solve:

$$\max_{I_t} E_t \sum_{\tau=t}^{\infty} \Lambda_{t,\tau}\Big\{Q_{s,\tau}I_\tau - \Big[1 + f\Big(\frac{I_\tau}{I_{\tau-1}}\Big)\Big]I_\tau\Big\}$$

Where $f\Big(\frac{I_t}{I_{t-1}}\Big) = \frac{\eta}{2}\Big(\frac{I_t}{I_{t-1}} - 1\Big)^2$ is the adjustment cost of net investment. The resulting selling price is given by

$$Q_{s,t} = 1 + \frac{\eta}{2}\Big(\frac{I_t}{I_{t-1}} - 1\Big)^2 + \frac{I_t}{I_{t-1}}\eta\Big(\frac{I_t}{I_{t-1}} - 1\Big) - E_t\Lambda_{t,t+1}\Big(\frac{I_{t+1}}{I_t}\Big)^2\eta\Big(\frac{I_{t+1}}{I_t} - 1\Big)$$

4.10 Retail firms

Retail firms, in a similar vein to households, are a unit continuum where a firm f belongs to the Core region if $f \in [0, n)$, these firms are given the marker h, and the foreign region if $f \in [n, 1]$. We focus on retail firms in the Core region but keep

[11] Investment is done on a per-region basis and therefore only Core capital producers can invest to produce capital used in Core production, there is no trade in capital or cross-country investment in this model.

the generic identifier f for firms to derive the problem of the retailer. The retailer bundles (CES aggregator) intermediate output at purchasing cost $P_{m,t}$ and sells it at price $P_t(f)$ as a final good for consumption. The CES aggregator of output in the Core region and Periphery can be written as:

$$Y_t = \left(\left(\frac{1}{n}\right)^{\frac{1}{\epsilon}} \int_0^n y_t(f)^{\frac{\epsilon-1}{\epsilon}} df \right)^{\frac{\epsilon}{\epsilon-1}} \quad Y_t^* = \left(\left(\frac{1}{1-n}\right)^{\frac{1}{\epsilon}} \int_n^1 y_t(f)^{\frac{\epsilon-1}{\epsilon}} df \right)^{\frac{\epsilon}{\epsilon-1}}$$

The demand function for goods produced by the individual firm is derived using the standard Dixit-Stiglitz problem, and the demand for the final good (as a share of total demand in that region) produced by firm f in the Core region depends on the relative price.

$$y_t(f) = \left(\frac{P_t(f)}{P_t}\right)^{-\epsilon} \frac{Y_t}{n} \quad \forall \; f \in [0, n)$$

A monopolistic retailer wishes to maximise profits π_t by choosing the price $P_t(f)$ to sell the final good, taking into account their input cost of intermediate goods and adjustment cost of prices à la Rotemberg (1982) :

$$\pi_t = P_t(f) y_t(f) - P^N_{m,t} y_t(f) - \frac{\psi}{2}\left(\frac{P_t(f)}{P_{t-1}(f)} - 1\right)^2 P_t Y_t$$

Dividing the profit by the price level and then taking the first order condition with respect to $P_t(f)$ gives us:

$$\begin{aligned} \frac{\partial}{\partial P_t(f)} &= (1-\epsilon)\left(\frac{P_t(f)}{p_t}\right)^{-\epsilon} \frac{Y_t}{nP_t} + \epsilon \frac{P^N_{m,t}}{P_t} \frac{1}{p_t} \left(\frac{P_t(f)}{p_t}\right)^{-\epsilon-1} \frac{Y_t}{n} \\ &- \psi \frac{1}{P_{t-1}(f)} \left(\frac{P_t(f)}{P_{t-1}(f)} - 1\right) \frac{p_t}{P_t} Y_t + E_t \Lambda_{t,t+1} \psi \frac{P_{t+1}(f)}{P_t(f)^2} \left(\frac{P_{t+1}(f)}{P_t(f)} - 1\right) \frac{p_{t+1}}{P_{t+1}} Y_{t+1} = 0 \end{aligned}$$

Since this is an identical problem for all firms within a region, each firm will choose the same price level, and $P_t(f) = p_t \;\; \forall \;\; f \in [0, n)$.[12] Price inflation of the final goods produced in the Core region is defined as $\Pi_{C,t} = \frac{p_t}{p_{t-1}}$. We can then rewrite the FOC as:

$$(1-\epsilon) + \epsilon P_{m,t} P_t \frac{1}{p_t} - \psi \Pi_{C,t}(\Pi_{C,t} - 1) + E_t \Lambda_{t,t+1} \psi \frac{\Pi^2_{C,t+1}}{\Pi_{t+1}} (\Pi_{C,t+1} - 1) \frac{Y_{t+1}}{Y_t} = 0$$

When $\psi = 0$ we are in a flexible price equilibrium and therefore price is set as a markup over marginal cost $\frac{P_{m,t} P_t}{p_t} = \frac{\epsilon - 1}{\epsilon}$. Due to the two-country setup there is a difference between producer price inflation $\Pi_{C,t}$ of goods produced in the Core region and consumer price inflation Π_t, which is the one faced by Core consumers and also takes into account their consumption of goods produced in the Periphery [13].

4.11 Closing the model

To close the model we state the resource constraint, a Fisher equation and the link between corporate securities and capital.

The resource constraint of output in the Core region is determined by total consumption of goods produced from the Core region[14], investment cost for the capital production firm, cost of asset purchases for the home region $\Phi_{C,t} = \tau(\psi_{s,t-1} Q_{s,t-1} S_{t-1} + \psi_{B,t} Q_{b,t-1} B_{t-1})$ and the adjustment cost paid by the retail firm to change their prices. Consumption of goods produced in the Core region is comprised of consumption of

[12] Recall that p_t represents the price of Core goods, this differs from P_t, which is a weighted sum of the price level faced by Core households and therefore also feature the price of Periphery produced goods consumed by Core households.

[13] In a standard one-country model where $\Pi_{C,t} = \Pi_t$ and $p_t = P_t$ the above equation reverts back to the standard Rotemberg (1982) pricing form with $P_{m,t}$ thought of as the real marginal cost.

[14] Where $c_t = \left(\frac{p_t}{P_t}\right)^{-\theta} \nu C_t$ and $c^*_t = \left(\frac{P^*_t}{P_t}\right)^{-\theta} (1-\nu) C_t$. ν represents home bias, and θ is the elasticity of substitution between goods in the Core and Periphery regions - see Appendix for further explanation.

Core goods by Core households, c_t, and consumption of Core goods by periphery households $c_{p,t}$.

$$Y_t = c_t + c_{P,t} + [1 + f\Big(\frac{I_t}{I_{t-1}}\Big)]I_t + \Phi_{C,t} + \frac{\psi}{2}(\Pi_{C,t} - 1)^2 Y_t$$

The total amount of corporate securities in the Core region is given by the investment conducted in that region by the capital producer and the remaining capital in the economy discounted at the standard rate δ. Since we are assuming that foreign banks cannot hold domestic region corporate securities this equation is identical to a one-country model equivalent.

$$S_t = I_t + (1 - \delta)K_t$$

Total output of the currency union, Y_t^U, is defined as the weighted sum of output from each region weighted by relative region size n.

$$Y_t^U = nY_t + (1 - n)Y_t^*$$

4.12 Calibration

The model is calibrated to the largest economies within the Eurozone and split into two regions, Core and Periphery. The Core is comprised of Germany, France and the Netherlands. The Periphery is comprised of Portugal, Italy, Ireland, Greece and Spain. The calibration seen in Table 1 focuses on 2012 and draws on national statistics, data from the IMF and household consumption bias from Bussière et al. (2013). Specifically, country size is set proportional to the Gross Domestic Product of each region. The statistic for home bias is taken from Bussière et al. (2013), who derive import contents of consumption up to 2005[15] for major world economies. We take the total debt over GDP, gross position for 2012, from the World Economic Outlook produced by the International Monetary Fund and use a weighted sum to arrive at 65.77% for the Core and 78.34% for the Periphery. Debt held by households and banks is calibrated on data by the European Central Bank. Lastly, the fraction of time spent working, which determines the steady state level of labour L and is chosen by adjusting, χ, the disutility of labour, is the weighted average of the number of people employed and hours worked in the Core and Periphery taken from the OECD. The rest of the calibration is standard and is drawn from Gertler and Karadi (2013) and Galí (2015). The effective lower bound is introduced into the paper using Occbin by Guerrieri and Iacoviello (2015). Occbin is a piecewise linear perturbation method that can handle occasionally binding constraints and is applicable to models with a large number of state variables.

[15] An underlying assumption is that import contents of consumption has been stable from 2005 to 2012 such that we are able to derive home bias from this statistic.

Table 1: Calibration

Description	Variable	**Core**	**Periphery**
	Model Specific		
Country Size	n	0.61	0.39
Home Bias in final goods	ν	0.77	0.77
Home bias in bonds (banking)	ν_b	0.81	0.61
Debt to GDP	by_t	65.77	78.34
Percent of Core debt held by households	$\bar{B}_h$	0.35	0.21
Percent of Periphery debt held by households	$\bar{B}_h^*$	0.13	0.49
Percent of core debt held by banks	b	0.33	0.11
Percent of periphery debt held by banks	b^*	0.12	0.27
Fraction of time spent working	L	0.24	0.30
	Conventional Parameters		
Capital share	α	0.36	0.36
Discount factor	β	0.9975	0.9975
Persistence of monetary policy decisions (Monetary union)	ϕ_i	0.5	0.5
Inflation feedback Taylor Rule (Monetary Union)	ϕ_π	2	2
Output feedback Taylor Rule (Monetary Union)	ϕ_y	0.125	0.125
Demand Elasticity	ϵ	3.857	3.857
Elasticity of labour supply	φ	2	2
Adjustment cost of Households holding bonds	κ	1	1
Absconding Rate	θ	0.3	0.3
Absconding for government bonds	Δ	0.7	0.7
Bankers startup fund	ω	0.0047	0.0047
Probability of banker survival	σ	0.95	0.917
Adjustment cost of investment	η	5.169	5.169
Adjustment cost for Rotemberg Pricing	ψ	34.03	34.03
Steady state inflation	Π_{ss}	1	1
Discount rate of capital	δ	0.025	0.025
Inefficiency of government purchases	τ	0.001	0.001
Elasticity of substitution between goods	θ_T	5	5
Elasticity of substitution between bonds (banking)	ι	1.1	1.1
Persistence of technology shock	ρ_a	0.9	0.9
Persistence of monetary policy shock	ρ_ν	0.9	0.9
Persistence of demand shock	ρ_ζ	0.9	0.9
Persistence of capital quality shock	ρ_ξ	0.7	0.7
Persistence of securities purchase shock	ρ_{st}	0.9	0.9
Persistence of bond purchase shock	ρ_{bt}	0.9	0.9

5 Results and policy simulations

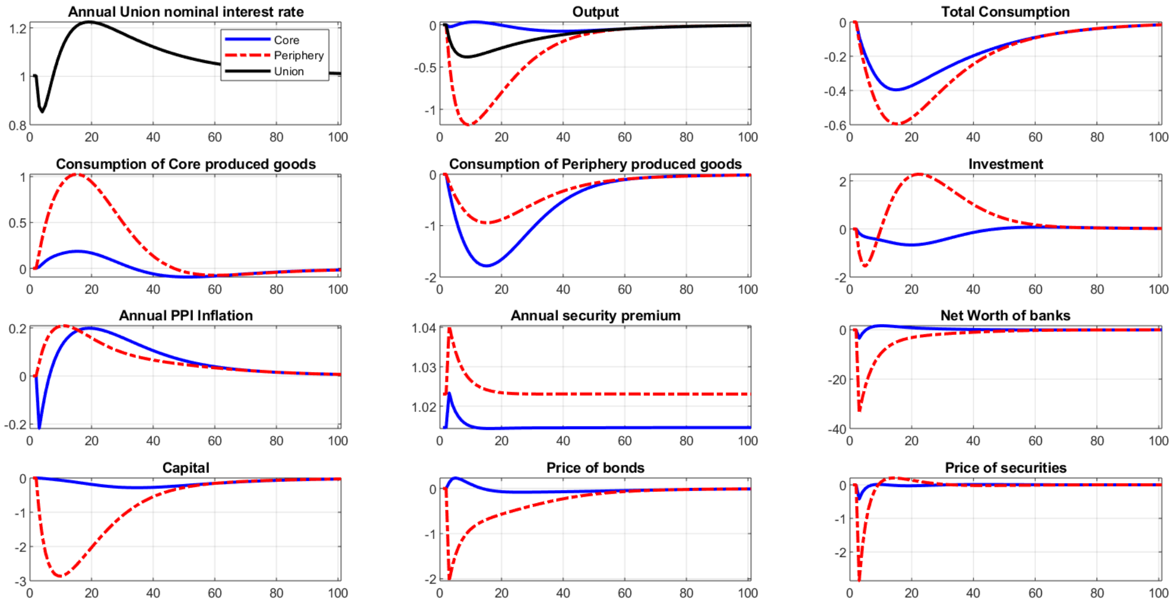
This section analyses the financial pass-through of a capital destruction shock from the Periphery region to the Core region. We explore the propagation of this shock under a fluid bond market compared to a rigid market. Moreover, the impact of the ELB on the real economy is detailed as well as the effect of government bond purchases and corporate security purchases within the monetary union.

5.1 Scenario 1: The role of bond market and financial pass-through

Figure 3 and 4 highlight the transmission through the banking sector of a one-percent-annualised capital destruction shock in the Periphery region. When government bonds are not easily substitutable, banks and households are less inclined to change their positions and the bond market is less fluid.

Figure 3 highlights the effect of the capital destruction shock and weak financial pass-through. With low elasticity of substitution the Periphery government bond premium and corporate security premium rises more, the value of Periphery government bonds and corporate securities fall more and Periphery investment is lower. Households from both regions increase their consumption of the relatively cheaper Core produced final goods, which helps to support Core output. The destruction of capital in the Periphery region also lowers the net worth of Periphery banks, causing them to sell off government bonds in order to adhere to their collateral constraint. Periphery banks sell Core and Periphery government bonds and households from both regions as well as the Core banks purchase these bonds.

Fig. 3: Capital destruction shock in Periphery with rigid bond market



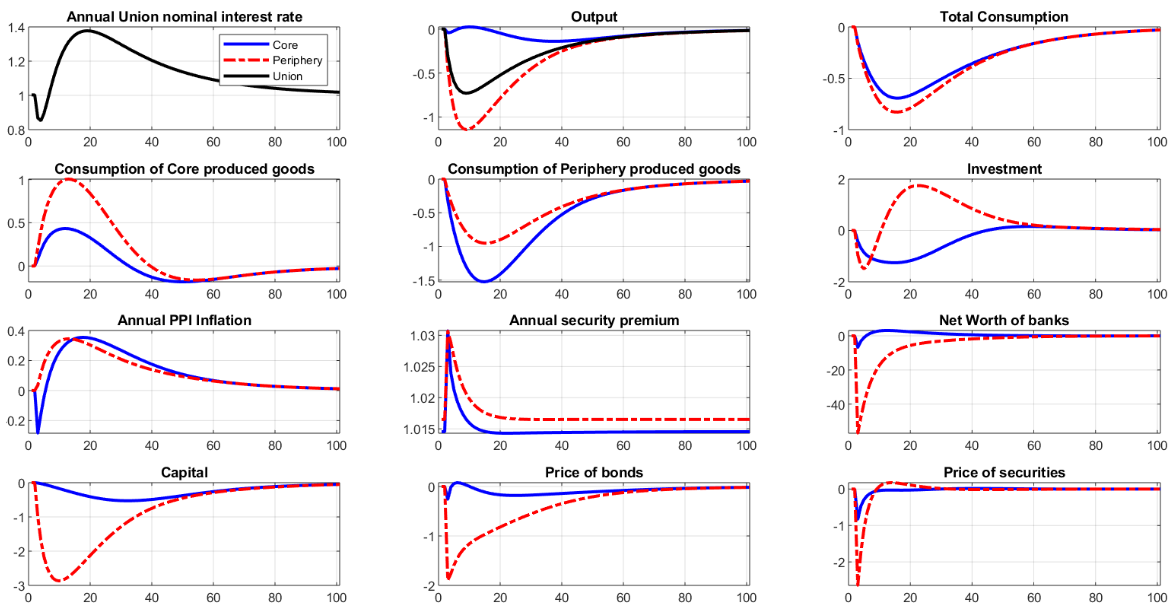
One-percent capital quality shock in the Periphery region. Bond elasticity $\iota = 1.1$ and household bond adjustment cost $\kappa = 1$

The main differences between Figure 3 and 4 can be seen in the role of the financial sector. When elasticity is high and households can easily trade government bonds, this capital destruction shock spreads across the monetary union. Both banks react more. Due to the higher fluidity of the bond market (primarily due to lowering the adjustment cost of government bonds for households κ), we see an unrealistic sell-off of Periphery government bonds. When the bond market was rigid the price of government bonds and corporate securities in the Core region rose, as demand for the safer Core government bonds increased. However, with higher pass-through we find that the shock spreads to the Core and the annual corporate security premium, as well as the annual government bond premium (not shown)[16], is much closer between Core and government bonds, signalling that the perceived risk of both regions is now

[16]The bond premium and security premium co-move as they are linked due to the collateral constraint on the banking sector.

similar. Moreover, we see lower investment in the Core region in Figure 4 compared to Figure 3 and a drop in consumption by the Core and Periphery households. When pass-through is high the central bank must react more to the capital destruction shock as the latter has a larger impact on the union as a whole, since it also affects the Core region.

Fig. 4: Capital destruction shock in Periphery with fluid bond market



One-percent annualised capital quality shock in the Periphery region. Bond elasticity $\iota = 100$ and $\kappa = 0.25$.

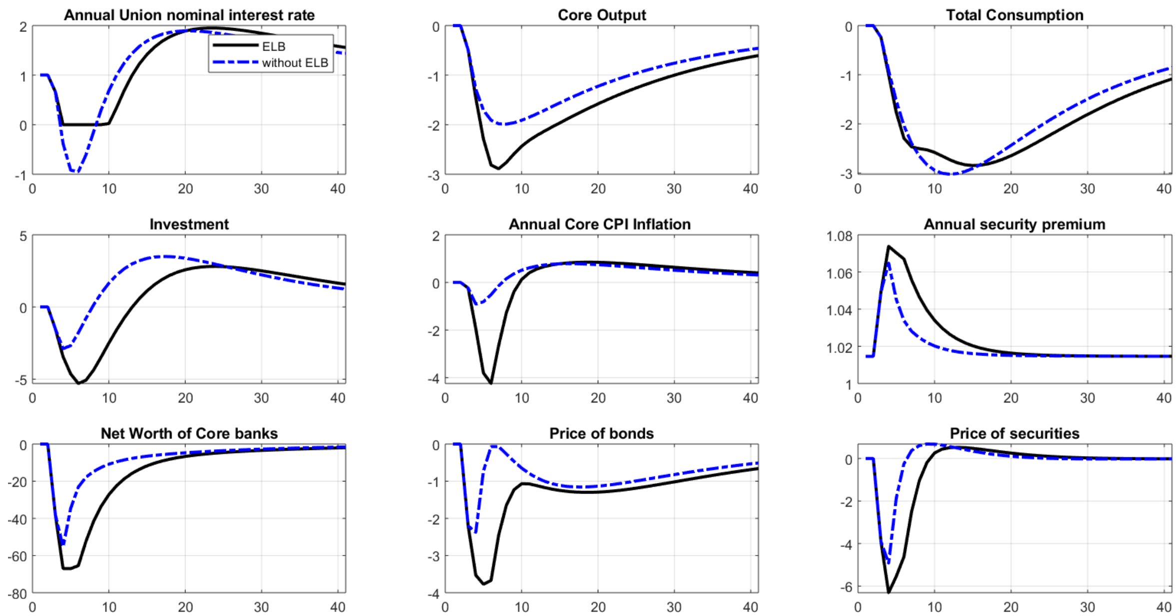
5.2 Scenario 2: Effective lower bound and Asset Purchases

Figure 5 displays the impact of reaching the effective lower bound from a series of capital destruction shocks and demand shocks[17] in both the Core and Periphery. The economy is at the effective lower bound for 6 quarters and restricting the central bank's ability to lower the interest rate negatively impacts consumption, output and

[17]Due to calibrating the model to 2012, and therefore the interest rate is set to 1%, it only takes two quarters of 1% capital destruction coupled with a 1.5% demand shock to both regions to reach the ELB.

inflation. In this scenario Core and Union-wide output is 1% lower than it otherwise would be if the central bank could lower the interest rate below zero and the CPI deflates by more than an additional 3%. Due to capital quality and negative demand shocks investment is reduced and the premium on both government bonds and corporate securities rise. Households, who wish to postpone consumption due to a falling demand, purchase government bonds from both Core and Periphery banks who are selling their bonds. The rebound in the price of government bonds one period after their fall is the result of this demand shock, which we assume impacts the model one period after the capital destruction shock. As previously shown, the capital destruction shock forces banks to sell government bonds on their balance sheet and their net worth falls. This effect is amplified if the interest rate, which is also the deposit rate, is held artificially high (as in the ELB), increasing the cost to the banks of household deposits.

Fig. 5: Effective lower bound in the Core region



Capital destruction and demand shocks in both regions force the monetary union to the ELB. The IRF focuses on the variables in the Core region. The effective lower bound is reached by a series of capital quality and then demand shocks.

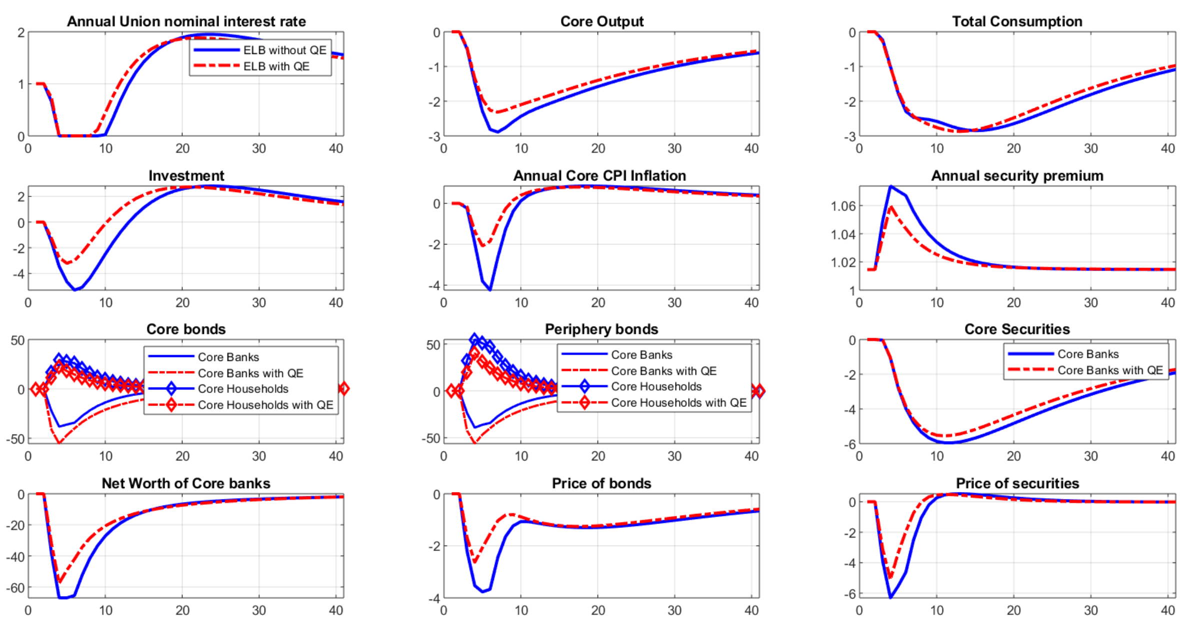
Figure 6 highlights the benefit of the central bank conducting government bond purchases. Bond purchases increase the price of bonds, supporting the banks and households balance sheet. These bond purchases support consumption and price inflation, allowing the central bank to escape the ELB earlier. The government bond purchases are calibrated to 10% of GDP in both the Core and Periphery regions for two quarters and then following an AR(1) process shown earlier. This process is set to be very persistent, however, not as severe as similar AR(2) processes. Due to modeling sovereign bonds a la Woodford (2001), and as such as a perpetuity with a decaying coupon, which have a maturity of 10 years, the price-quantity nexus of public debt is altered as shown in Auray and Eyquem (2017).[18] As is found in Auray and Eyquem

[18] Auray and Eyquem (2017) find that longer maturities are associated with low steady state bond levels but higher bond prices, such that debt-to-GDP is the same but the amount of goverment bonds purchased is reduced. This is why in figure 6 the quantity of bonds purchased by the monetary union

(2017) longer bond maturities dampen the movement in output and consumption. This may be a contributing factor in why government bond purchases have a smaller effect on the economy than seen in Gertler and Karadi (2013), which focuses on the US experience and has one-period government bonds.

Finally, government bond purchases help to support bond prices, which means that banks wish to sell more government bonds and purchase corporate securities, which offer a higher return. These purchases help to limit the rise in the annual government bond and corporate security premium for the Core region (shown in the figure) and Periphery region, representing a lowering of risk in the market. Investment and CPI inflation are also supported by this intervention.

Fig. 6: Effective Lower Bound and Bond Purchases (10% of GDP)



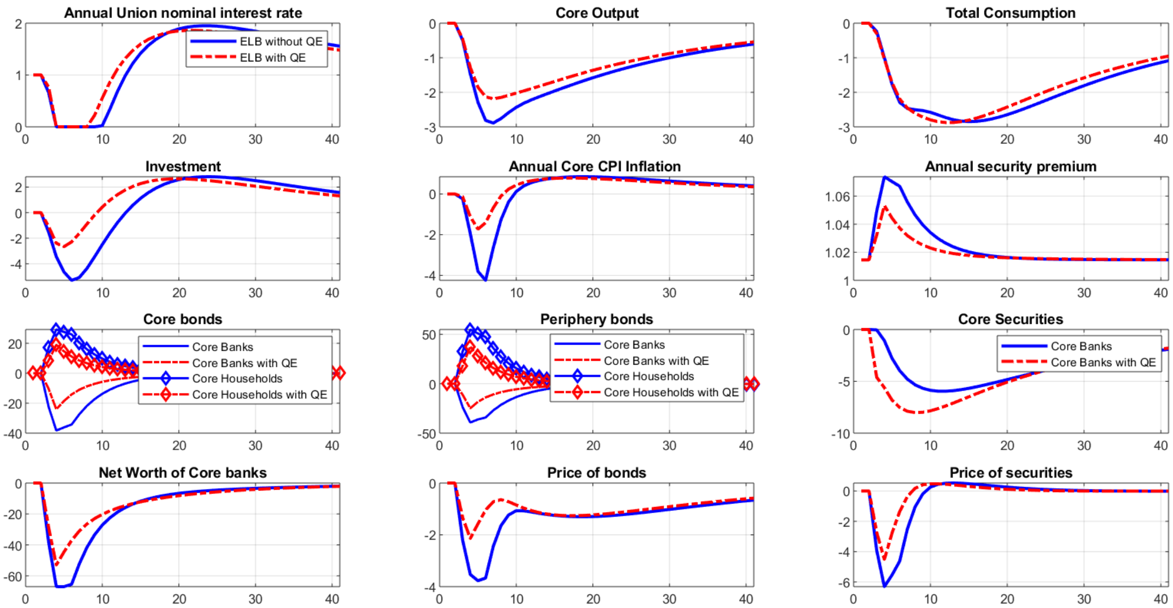
Figures show response of the Core region only. The effective lower bound is reached by a series of capital quality and then demand shocks. Bond purchase shock is calibrated to reach 10% of GDP for 2 quarters and follow an AR(1) process.

is minimal since we are assuming these bonds have a long duration and their steady-state price is high relative to a bond of one quarter duration.

The effect of corporate security purchases by the central bank can be seen in Figure 7. The impact on the economy from corporate security purchases is larger than that of government bond purchases, primarily due to the collateral constraint introduced on the banking sector and the higher risk of absconding with corporate securities than with government bonds. This assumption implies that, by buying corporate securities, the Central Bank obtains a larger relaxation of the incentive compatibility constraint, and a larger impact on the economy. The transmission channel from the corporate security purchase shock seen in Figure 7 is through the portfolio re-balance channel as banks move back into government bonds and households sell bonds to deposit into banks, thereby also benefiting from the asset purchases. The main difference between Figure 6 and Figure 7 can be seen in the response of banks as when the Central Bank conducts bond purchases the banks sell more bonds and move into corporate securities whereas when the Central Bank purchases corporate securities the banks sell more corporate securities and hold more government bonds.[19]

[19] Additional results can be found in Appendix D where the impact of asset purchases in the Periphery versus Core on union-wide output and inflation are assessed.

Fig. 7: Effective Lower Bound and Security Purchases (10% of GDP)



The figure focuses on the response of Core region variables. The effective lower bound is reached by a series of capital quality and then demand shocks. The corporate security purchase shock is calibrated to reach 10% of GDP for the first two quarters and follow an AR(1) process.

6 Closing remarks

This paper explores the effectiveness of government bond and corporate security purchases by a central bank within a calibrated two-country New-Keynesian model featuring a banking sector (an extension of Gertler and Karadi (2011) and Andrade et al. (2016)) and a two-country monetary union. It further explores the propagation of economic shocks from one region to another in a monetary union and how these propagation depends on the banking sector.

We propose a rich setup, where households also hold government bonds and capital is region-restricted, and we account for the maturity effect of longer-term government bonds. We find that a negative (capital destruction) shock in the Periphery causes a fall in the output of the entire union, and this propagation is amplified if financial markets are integrated so that banks and households can freely trade government bonds. The impact on the financial economy is always sizeable, even with a small adjustment to the fluidity of the government bond market.

Our second finding concerns the effect of non-conventional monetary policy, as we focus on the effectiveness of government bond purchases versus corporate security purchases. As in Gertler and Karadi (2013) we analyse the impact of government bond purchases versus corporate security purchases at the effective lower bound. Due to the nature of the collateral constraint we find that corporate security purchases have a stronger impact on inflation and on lift-off time from the Effective Lower Bound than equivalent government bond purchases. This finding is in line with the ones of Gertler and Karadi (2013) for the U.S. economy. However, the large difference, seen in Gertler and Karadi (2013), between these two quantitative easing policies is not present in our model. This is likely due to our calibration to the eurozone, which has a smaller share of corporate securities than the U.S.

There are many experiments that can be done with our model setup: analysing the propagation of different shocks and the effect of different degrees of symmetry among them, testing a more realistic QE schedule (AR(2) rather than AR(1)), adjusting the size and stability of a region to see how this impacts the monetary union. These are left for future work.

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Appendix A CES aggregator and Dixit-Stiglitz

We use the CES formulation for consumption, prices and bond holdings of private banks in our model. As is standard in the literature we also derive the consumption of domestic and imported goods as a share of total consumption using the Dixit-Stiglitz aggregator, from Dixit and Stiglitz (1977).

Focusing on the Core region, since the problem is symmetric, we define aggregate consumption C_t in the Core region as the share of domestic consumption c_t and foreign consumption c_t^*, taking into account home bias in consumption $\nu \in [0,1]$ and the elasticity of substitution, θ, between Core and Periphery goods:

$$C_t \equiv \left[(\nu)^{\frac{1}{\theta}}(c_t)^{\frac{\theta-1}{\theta}} + (1-\nu)^{\frac{1}{\theta}}(c_t^*)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} \tag{A.1}$$

The price index in the Core region follows a similar form to the consumption CES. It combines the price of Core goods p_t and the price of Periphery goods p_t^*, weighted by the respective amount the representative Core consumer would purchase.

$$P_t \equiv \left[\nu(p_t)^{1-\theta} + (1-\nu)(p_t^*)^{1-\theta}\right]^{\frac{1}{1-\theta}}$$

Using the standard Dixit-Stiglitz miniminsation it is possible to derive the the demand functions c_t and c_t^* as a share of total consumption in the Core region weighted by the relative price of the goods and the home bias:

$$c_t = \left(\frac{p_t}{P_t}\right)^{-\theta} \nu C_t$$

$$c_t^* = \left(\frac{p_t^*}{P_t}\right)^{-\theta} (1-\nu) C_t$$

Allowing ϵ to be defined as the elasticity of substitution across the differentiated goods within a region. Therefore it is possible to derive the price aggregators for goods originating in the Core and Periphery regions by minimising the cost of each bundle, taking the prices of the differentiated goods as given. The result is:

$$p_t \equiv \left[\frac{1}{n} \int_0^n p_t(c)^{\frac{\epsilon-1}{\epsilon}} dc \right]^{\frac{\epsilon}{\epsilon-1}}$$

$$p_t^* \equiv \left[\frac{1}{1-n} \int_n^1 p_t(p)^{\frac{\epsilon-1}{\epsilon}} dp \right]^{\frac{\epsilon}{\epsilon-1}}$$

The second stage of the Dixit-Stiglitz aggregator allows us to derive the consumption of each good consumed in the Core region, c_t and c_t^*, as an index of consumption across the continuum of differentiated goods. Where the size of the Core region is denoted as n.

$$c_t \equiv \left[\left(\frac{1}{n} \right)^{\frac{1}{\epsilon}} \int_0^n c_t(c)^{\frac{\epsilon-1}{\epsilon}} dc \right]^{\frac{\epsilon}{\epsilon-1}}$$

$$c_t^* \equiv \left[\left(\frac{1}{1-n} \right)^{\frac{1}{\epsilon}} \int_n^1 c_t(p)^{\frac{\epsilon-1}{\epsilon}} dp \right]^{\frac{\epsilon}{\epsilon-1}}$$

$$c_t(c) = \left(\frac{P_t(c)}{p_t} \right)^{-\epsilon} \frac{c_t}{n}$$

$$c_t(p) = \left(\frac{P_t(p)}{p_t^*} \right)^{-\epsilon} \frac{c_t^*}{1-n}$$

Combining the demand and price equation allows us to rewrite consumption in the Core region by the differentiated goods c and p.

$$\Rightarrow c_t(c) = \left(\frac{p_t(c)}{p_t} \right)^{-\epsilon} \frac{\nu}{n} \left(\frac{p_t}{P_t} \right)^{-\theta} C_t$$

$$\Rightarrow c_t(p) = \left(\frac{p_t(p)}{p_t^*}\right)^{-\epsilon} \frac{1-\nu}{1-n} \left(\frac{p_t^*}{P_t}\right)^{-\theta} C_t$$

Appendix B Relation of prices in the model

It is useful to have the relation of prices explicitly written out as they are used extensively in multi-country models. .

Terms of trade (in producer pricing):

$$T_t = \frac{p_t^*}{p_t}$$

Harmonised index of consumer prices:

$$\Pi_{t+1}^U = \frac{P_{t+1}^U}{P_t^U}$$

Consumer Price Inflation in Region H:

$$\Pi_{t+1} = \frac{P_{t+1}}{P_t}$$

Harmonised level of consumer prices:

$$P_t^U = \left(P_t\right)^n \left(P_t^*\right)^{1-n}$$

Price of consumption by households:

$$P_t \equiv \left[\nu\left(p_t\right)^{1-\theta} + (1-\nu)\left(p_t^*\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}$$

Further price relations:

$$\frac{P_t}{p_t} = \left[\nu\left(\frac{p_t}{p_t}\right)^{1-\theta} + (1-\nu)\left(\frac{p_t^*}{p_t}\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}$$

$$\Rightarrow \frac{P_t}{p_t} = \left[\nu + (1-\nu)T_t^{1-\theta}\right]^{\frac{1}{1-\theta}}$$

$$\frac{P_t^*}{p_t^*} = \left[\nu^* + (1-\nu^*)T_t^{\theta-1}\right]^{\frac{1}{1-\theta}}$$

$$\frac{P_t}{p_t^*} = \left[\nu T_t^{\theta-1} + (1-\nu)\right]^{\frac{1}{1-\theta}}$$

$$\frac{P_t^*}{p_t} = \left[\nu^* T_t^{1-\theta} + (1-\nu^*)\right]^{\frac{1}{1-\theta}}$$

$$\frac{P_t^U}{P_t} = \left[\frac{\nu^* T_t^{1-\theta} + (1-\nu^*)}{\nu + (1-\nu)T_t^{1-\theta}}\right]^{\frac{1-n}{1-\theta}}$$

$$\frac{P_t^U}{P_t^*} = \left[\frac{\nu T_t^{\theta-1} + (1-\nu)}{\nu^* + (1-\nu^*)T_t^{\theta-1}}\right]^{\frac{n}{1-\theta}}$$

$$\frac{P_t^*}{P_t} = \left[\frac{\nu^* T_t^{1-\theta} + (1-\nu^*)}{\nu + (1-\nu)T_t^{1-\theta}}\right]^{\frac{1}{1-\theta}}$$

Change in the terms of trade is the change in price inflation of the Periphery produced goods compared to Core, (PPI inflation).

$$\frac{T_t}{T_{t-1}} = \frac{\Pi_{P,t}}{\Pi_{C,t}}$$

Where $\Pi_{P,t}$ is the PPI of Periphery produced goods and $\Pi_{C,t}$ is the PPI of the Core produced goods.

Appendix C Two country bank problem derivation

This section derives the banker's problem seen in the main text Section 4.3. We focus on the Core banker and assume that it is equally difficult to abscond with domestic government bonds as it is to abscond with Periphery government bonds within a monetary union. For ease of reference we outline again the CES form that the bond holdings of the banker takes:

$$b_{CES,t} = \left(\nu_b^{\frac{1}{\iota}} b_t^{\frac{\iota-1}{\iota}} + (1-\nu_b)^{\frac{1}{\iota}} b_t^{*\frac{\iota-1}{\iota}} \right)^{\frac{\iota}{\iota-1}}$$

$$Q_{b,CES,t} b_{CES,t} = Q_{b,t} b_t + Q_{b,t}^* b_t^*$$

Using this CES structure allows us to solve the bankers problem in an analogous manner to the one-country bank model. Therefore we set up two value functions, the end of period value function V_{t-1} and the beginning of next period value function W_t. As before the value of a bank at the end of the period is equal to the franchise value of the bank with assets: $s_{t-1}, b_{CES,t-1}, n_{t-1}$. We write this by equating the beginning of next periods value taking into account the survival probability σ:

$$V_{t-1}(s_{t-1}, b_{CES,t}, n_{t-1}) = E_{t-1} \Lambda_{t-1,t} \{(1-\sigma) n_t + \sigma W_t(n_t)\}$$

To solve this problem first conjecture that the value function is linear in state variables with the coefficients: $\mu_{s,t}, \mu_{b,t}$, to be determined.

$$V_t = \mu_{s,t} Q_{s,t} s_t + \mu_{b,t} q_{b,CES,t} b_{CES,t} + v_t n_t$$

The banks problem is to select assets, $s_t, b_{CES,t}$, to maximise its net worth while still respecting the collateral constraint (incentive compatibility constraint) imposed by the households. The collateral constraint enforces a limit on the leverage ratio of the bank.

$$W_t(n_t) = \max_{s_t, b_{CES,t}} V_t(s_t, b_{CES,t}, n_t)$$

Subject to:

$$V_t(s_t, b_{CES,t}, n_t) \geq \theta Q_{s,t} s_t + \Delta\theta(q_{b,CES,t} b_{CES,t})$$

The above problem can be rewritten using the incentive compatibility constraint with a lagrange multiplier λ_t associated with this constraint:

$$\max \;\; V_t(\cdot) + \lambda_t(V_t(\cdot) - \theta Q_{s,t} s_t + \Delta\theta q_{b,CES,t} b_{CES,t})$$

$$\Rightarrow (1+\lambda_t)V_t(\cdot) - \lambda_t(\theta Q_{s,t} s_t + \Delta\theta q_{b,CES,t} b_{CES,t})$$

Writing out the equation to maximise can be summarized as:

$$(1+\lambda_t)[\mu_{s,t} Q_{s,t} s_t + \mu_{b,t} q_{b,CES,t} b_{CES,t} + v_t n_t] - \lambda_t(\theta Q_{s,t} s_t + \Delta\theta q_{b,CES,t} b_{CES,t})$$

The first order conditions are therefore:

$$\begin{aligned}
\frac{\partial}{\partial s_t} &= (1+\lambda_t)\mu_{s,t} Q_{s,t} = \lambda_t \theta Q_{s,t} \\
&\Rightarrow \mu_{s,t} = \frac{\lambda_t}{1+\lambda_t}\theta \\
\frac{\partial}{\partial b_{CES,t}} &= \mu_{b,t} q_{CES,t}(1+\lambda_t) = \lambda_t \Delta\theta q_{b,CES,t} \\
&\Rightarrow \mu_{b,t} = \frac{\lambda_t}{1+\lambda_t}\Delta\theta = \Delta\mu_{s,t}
\end{aligned}$$

The complementary slackness condition (lagrange multiplier times the constraint) is written below. It must hold that either the constraint is binding and therefore the lagrange multiplier is non-zero (positive) or the constraint does not bind and the lagrange multiplier λ_t is zero.

$$\lambda_t[\mu_{s,t}Q_{s,t}s_t + \mu_{b,t}q_{b,CES,t}b_{CES,t} + v_t n_t - (\theta Q_{s,t}s_t + \Delta\theta q_{b,CES,t}b_{CES,t})] = 0$$

Since we are assuming that the constraint binds with equality it must be that the terms inside the bracket are zero, therefore using the complementary slackness condition we can write:

$$\Rightarrow \mu_{s,t}Q_{s,t}s_t + \mu_{b,t}q_{b,CES,t}b_{CES,t} + v_t n_t = \theta Q_{s,t}s_t + \Delta\theta q_{b,CES,t}b_{CES,t}$$

Rewriting the constraint in terms of the net worth n_t :

$$\Rightarrow v_t n_t = (\theta - \mu_{s,t})Q_{s,t}s_t + \Delta(\theta - \mu_{s,t})q_{b,CES,t}b_{CES,t}$$

$$\Rightarrow v_t n_t = (\theta - \mu_{s,t})[Q_{s,t}s_t + \Delta q_{b,CES,t}b_{CES,t}$$

$$\frac{v_t}{\theta - \mu_{s,t}} n_t = Q_{s,t}s_t + \Delta q_{b,CES,t}b_{CES,t}$$

$$\phi_t n_t = Q_{s,t}s_t + \Delta q_{b,CES,t}b_{CES,t}$$

Where

$$\phi_t = \frac{v_t}{\theta - \mu_{s,t}} \quad \text{Leverage Ratio in the Core region}$$

The leverage ratio, ϕ_t, is the maximum value of assets over net worth that the banker can hold without violating its incentive compatibility constraint. If the incentive constraint binds (assumed it does) then this is the leverage of the bank.

The beginning of period value function W_t is also linear and can be written as a function of the net worth of the banker:

$$
\begin{aligned}
W_t(n_t) &= \mu_{s,t}(Q_{s,t}s_t + \Delta q_{b,CES,t}b_{CES,t} + v_t n_t \\
&= (\mu_{s,t}\phi_t + v_t)n_t \\
&= \theta\phi_t n_t
\end{aligned}
$$

Using the beginning of period value function we can derive the end of period value function by rewriting V_{t-1} and inserting $W_t(n_t)$:

$$\mu_{s,t-1}Q_{s,t-1}s_{t-1} + \mu_{b,t-1}q_{b,CES,t-1}b_{CES,t-1} + v_t n_{t-1} = E_{t-1}\Lambda_{t-1,t}\{(1-\sigma)n_t + \sigma W_t(n_t)\}$$

The flow of funds for the banker is given by the returns on holding securities and the returns on holding Core and Periphery bonds minus the interest payments that the banker owes the households for their deposits.

$$n_t = R_{Hk,t}Q_{Hs,t-1}s_{b,t-1} + R_{b,CES,t}Q_{b,CES,t-1}b_{CES,_{t-1}} - R_{H,t}d_{t-1}$$

Net worth develops as the benefit of holding claims on non-financial firms s_{t-1}, which is $(R_{K,t} - R_t)Q_{s,t-1}$ and the benefit of holding government bonds plus the previous periods net worth.

$$n_t = (R_{K,t} - R_t)Q_{s,t-1}s_{t-1} + (R_{b,CES,t} - R_t)q_{b,CES,t-1}b_{CES,t-1} + R_t n_{t-1}$$

Using what we have derived above it is possible to find the values of the coefficients $\mu_{s,t}, \mu_{b,t}$ of our linear value function:

$$\begin{aligned}\mu_{s,t-1}Q_{s,t-1}s_{t-1} + \mu_{b,t-1}q_{b,CES,t-1}b_{CES,t-1} + v_t n_{t-1} \\ = E_{t-1}\Lambda_{t-1,t}\{[(1-\sigma) + \sigma\theta\phi_t](R_{K,t} - R_t)Q_{s,t-1}s_{t-1} \\ + (R_{b,CES,t} - R_t)q_{b,CES,t-1}b_{CES,t-1} + R_t n_{t-1}]\}\end{aligned}$$

Which therefore means that:

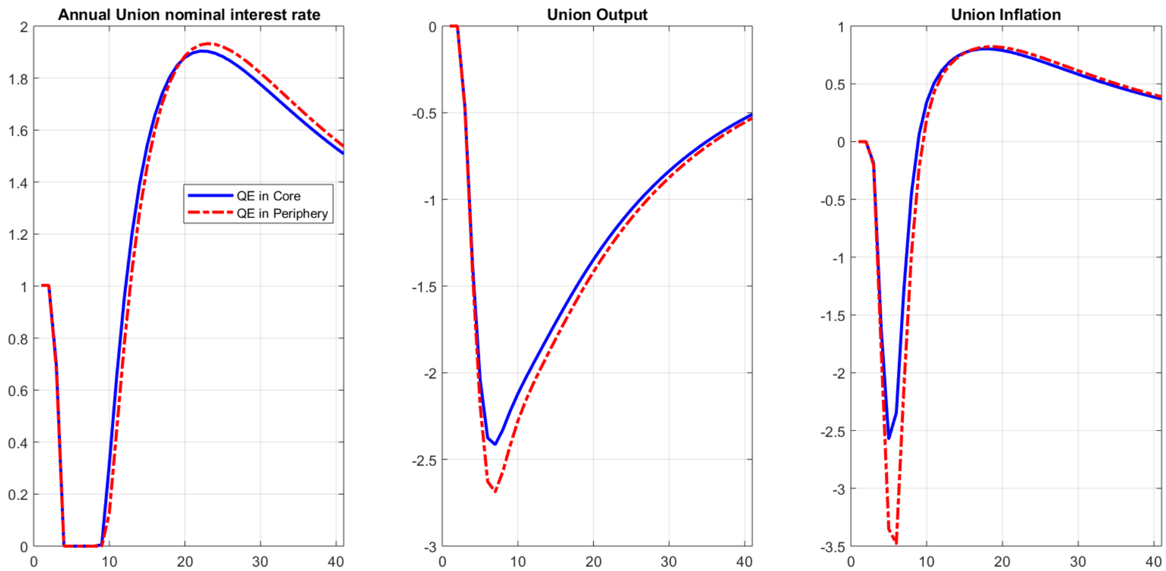
$$\begin{aligned}\mu_{s,t} &= E_t\Omega_{t,t+1}(R_{k,t+1} - R_{t+1}) \\ \mu_{b,t} &= E_t\Omega_{t,t+1}(R_{b,CES,t+1} - R_{t+1}) \\ v_t &= E_t\Omega_{t,t+1}R_{t+1} \\ \Omega_{t-1,t} &= \Lambda_{t-1,t}[1 - \sigma + \sigma\theta\phi_t]\end{aligned}$$

The bank's stochastic discount factor is given by $\Omega_{t-1,t}$, this is derived from the household's discount factor but additionally takes into account the probability of the banker exiting, thus it is augmented by the shadow value of unit of the net worth of the bank. This reflects the benefit of holding a larger amount of net worth, allowing the banker to retain more assets (whilst respecting the leverage restraint), and forms a crucial part of the financial accelerator mechanism. The amount of assets a bank is allowed to hold is partly determined by the absconding rate θ. The above solution shows that the end of period value function for the banker is linear and the coefficients of this are independent of bank specific variables. This means that it is possible to aggregate the banking sector and solve the model as if there was only one large bank (a representative bank) or a multitude of identical banks per region. The aggregation (which is used in the computation) can be found in the main text.

Appendix D Asset Purchases in Core or Periphery

This section outlines the impact of bond purchases by the monetary union in either the Core or Periphery region. Figure D.1 displays the result of the monetary union purchasing bonds worth 10% of GDP in the Periphery region compared to the equivalent value of bonds purchased in the Core region. Purchasing bonds in the Core region helps to support output and inflation by more than in the Periphery. It is important to stress that this finding is partly due to assuming the Core region has a higher level of technology, A_t, and therefore supporting production in the Core region is more beneficial than supporting production in the Periphery. Moreover, since households are able to freely borrow and save their marginal propensities to consume across the monetary union are similar, which may not be the case in reality. For instance, if a greater share of households act hand-to-mouth in the Periphery then monetary stimulus would lead the households in the Periphery to increase their consumption by more than households in the Core and therefore support output by more.

Fig. D.1: Bond Purchases in Core versus Periphery



The effective lower bound is reached by a series of capital quality and then demand shocks. The bond purchase shock is calibrated to reach 10% of GDP for the first two quarters in the Periphery region and equivalent value for the Core. The bond purchases follow an AR(1) process.