Optimal Monetary Rules with Downward Nominal Wage Rigidity*

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Abstract

At the individual and country level nominal wages have been found to be downwardly rigid, such that they are more likely to increase than decrease. This has strong implications for optimal monetary policy in the standard New-Keynesian model, which typically assumes flexible wages or symmetric nominal wage rigidities. This constraint causes the optimal monetary policy to react asymmetrically to symmetric shocks. Furthermore, motivated by the welfare loss generated by using a standard Taylor rule, this paper searches for a new optimal simple rule that can replicate the optimal monetary policy in this framework. As an extension I solve a non-linear model that internalises this constraint at all periods in time, which dampens wage increases in a model where agents can flexibly increase their wage, thus creating an endogenous rigidity. This work adds to the literature by introducing the downward nominal wage rigidity (DNWR) constraint of Schmitt-Grohé and Uribe (2016) into a standard New-Keynesian model and finds an optimal simple rule that places a high weight on the unemployment gap. Moreover, as with other work on DNWR, this paper finds support for 'greasing the wheels' - positive trend inflation that helps to deflate real wage increases.

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1 Introduction

In the standard New-Keynesian model wages are assumed to be symmetrically flexible, such that wage increases or decreases are equally effortless to implement. Extensions of this simple model, such as a medium-scale dynamic stochastic general equilibrium model, add Calvo wages,[1] which symmetrically dampens wage changes. However, as shown in the data of individuals' wage changes in Daly et al. (2012) and at the country level for developed and developing countries by Schmitt-Grohé and Uribe (2016), wages are downwardly rigid - we observe increases in nominal wages more often than decreases. Adding in such a constraint into a standard model impacts how the agents in the economy react to shocks, which has further consequences for the optimal monetary policy and the optimal steady state inflation rate compared to a model with flexible wages.

This paper explores monetary policy in the New Keynesian model when the model is affected by downward nominal wage rigidities (DNWR), such that nominal wages can freely rise but are sluggish when adjusting downwards. Including DNWR, instead of Calvo wages or flexible wages, causes an asymmetric response of monetary policy to shocks of the same size but differing signs. Moreover, shocks that increase the nominal wage, such as a temporary positive demand shock, can create persistent effects since the wage cannot adjust down in a timely manner, which causes an increase in unemployment as firms cannot afford to hire the workers but the artificially higher wage incentivises many households to enter the workforce, leading to a boom-bust cycle. This isn't found when Calvo wages are assumed. Furthermore, I find that the optimal monetary policy is asymmetric and allows for price inflation to deflate real wages (both of which have been found using a related framework in Kim and Ruge-Murcia (2011)). This work contributes to the literature by finding the optimal simple rule (OSR) in a New-Keynesian model with a downward nominal wage rigidity constraint of the form in Schmitt-Grohé and Uribe (2016). This optimal simple rule, referred to in the paper as Simple Rule 3, places a higher weight on deviations from the natural rate of unemployment than the typical Taylor rule,

[1] Wages are set by labour unions that are able to adjust the wage with a certain probability, defined as Calvo wages following Calvo (1983).

referred to as Simple Rule 1 or the simple rule seen in Galí (2015), referred to as Simple Rule 2. As in, Schmitt-Grohé and Uribe (2016), when the economy su ers from unemployment the labor unions wish to lower wages, however, due to the DNWR constraint this isn't feasible, thus motivating an interest rate rule that is more sensitive to unemployment changes. In the latter part of the paper, optimal trend in ation of around 0% to 1.25%, depending on the shocks assumed, is found to be welfare improving, thus providing further support for `greasing the wheels' of the economy. In an extension of the model, the downward nominal wage rigidity constraint is internalised, such that households optimise over it. Once the households understand the existence of this occasionally binding constraint, and that a shock may cause it to bind, it leads them to limit their wage increases even though the household can exibly raise the wage, this is because once increased the wage is sluggish to fall, a nding made as well by Elsby (2009) and more recently in a theoretical model by Wolf (2018).

The paper is organised as follows: Section 2 highlights the empirical motivation. Section 3 discusses the papers that have implemented a downward nominal wage constraint and outlines their ndings. Section 4 outlines the simple model used, which is a log-linearised New Keynesian model with an exogenous DNWR constraint or DNWR wage setting rule. Section 5 provides the impulse responses from simulating the model and presents the optimal monetary policy and optimal simple rule in this setup. Section 6 solves the non-linear model with a positive trend in ation and internalised DNWR constraint, allowing the households to be forward looking when setting their wages. Section 7 concludes and the Appendix includes details on the model equations, steady state, computational techniques used and additional tables and gures.

2 Empirical Motivation

This paper utilizes the basic New Keynesian model, which includes sticky prices in the standard form of Calvo (1983). The empirical motivation of this type of model has already been covered extensively. Therefore this section aims to empirically motivate the non-standard additions to the New Keynesian model that are used within this paper.

Previous work has been completed concerning wage rigidities, stemming from Erceg et al. (2000), which enter in symmetrically and are analogous to price rigidities. This setting has also been extended to include unemployment[2], originating from Galí (1996) and worked upon in Galí (2011) and Galí et al. (2011). The principle nding of which, the structural wage equation derived here is shown to account reasonably well for the co-movement of wage in ation and the unemployment rate in the US economy, Galí (2011).

However, there exists empirical evidence that wages may not be symmetrically rigid and that for developed countries, as well as some developing countries, wages are downwardly rigid - such that we observe increases in nominal wages more often than decreases, Schmitt-Grohé and Uribe (2016). The parameters of the model included in this paper are based upon U.S. data and hence the evidence for DNWR presented here is U.S. focused. The three main empirical studies that use individual level data are: Gottschalk (2005), Barattieri et al. (2014) and Daly et al. (2012). Gottschalk (2005) applies new methods to data from the Survey of Income and Program Participation and nds that downward exible wages found in individual level data is due to measurement error and once corrected the data produce ndings closer to that found in rm level data where only 2% to 3% of workers experiencing nominal-wage cuts, which implies substantial rigidity. In keeping with Gottschalk (2005), Barattieri et al. (2014) nds evidence using the same micro data that wage changes are signi cantly right-skewed therefore seeing an increase in wages is more likely than a decrease. Moreover, they also show that

> higher wage stickiness makes it easier for macroeconomic models to match the stylized fact that monetary shocks cause persistent changes in real output and small but relatively persistent changes in prices.(Barattieri et al., 2014)

Lastly Daly et al. (2012) analyses wage growth during the great recession of 2007 using the Current Population Survey and nds that despite modest economic growth and persistently high unemployment, real wage growth has averaged 1.1% since 2008 and that a signi cant fraction of workers are a ected by downward

[2] Staggered wage contracts and their link to unemployment can be seen even earlier in a rational expectations model of Taylor (1980).

nominal wage rigidities. One possible reason for observing DNWR during that period is that the low inflation environment meant that real wages were not being eroded by inflation. Furthermore, employers are hesitant to reduce pay as it can reduce morale and prompt resistance Kahneman et al. (1986).

Further empirical support of downward nominal wage rigidities has been found for European countries by the Wage Dynamics Network, a research network consisting of the European Central Bank (ECB) and the National Central Banks (NCBs) of the EU Member States. Using a firm-level survey spanning 15 European countries during the late 2007 and early 2008, Babecky et al. (2010) find evidence of downward nominal wage rigidity (defined in their study as wage freezes) and downward real wage rigidity (defined through wage indexation). Further evidence is provided by research conducted by the Wage Dynamics Network, which focuses on downward real wage rigidity for specific countries over a longer period of time, including Lunnemann and Wintr (2010) who focuses on Luxembourg between 2001 and 2007, and Du Caju et al. (2012) for Belgium between 1990 and 2002.

However recent work by Elsby et al. (2016), which focuses on the USA and UK labour markets, argue that downward nominal wage rigidity may be less binding than originally thought. Through the use of higher quality data - payroll data instead of self-reported surveys that may be subject to reporting error - and a comparison between male and female workers they find a higher frequency of wage reductions than previous studies. Thus motivating further empirical research into the existence and impact of downward nominal wage rigidities.[3]

3 Literature Review

Downward Nominal Wage Rigidites have been studied within economic models previously. The innovation within this paper is to include it in the New Keynesian model with unemployment using a simple constraint, exploring the optimal monetary policy and a proposed optimal simple rule. Below provides an outline and briefly discusses the most prominent papers within the literature that

[3]For example, Elsby and Solon (2019) present international evidence on the frequency of wages cuts.

include DNWR.

Kim and Ruge-Murcia (2011), which builds on one of their earlier papers[4] utilize a convex cost function for changing prices and wages that can be asymmetric or reduced down to a quadratic cost a la Rotemberg (1982). Moreover, this cost function encompasses the `L' shaped cost function of Benigno and Ricci (2011) which corresponds to the situation where cutting wages is infinitely costly and raising wages is costless. They find that `greasing the wheels', having a low but strictly positive inflation target is welfare improving. Therefore,

> for an economy with downwardly rigid wages, the benefits of positive inflation conjectured by Tobin (1972) may overcome Friedman (1969)'s general prescription of negative inflation.(Kim and Ruge-Murcia, 2011)

This inflation target is estimated to be around 1% but will change depending on the model specifications and country estimated to. Kim and Ruge-Murcia (2011)'s paper is similar to this paper in execution and conclusion however this model utilizes fully flexible increases in nominal wages with DNWR and Calvo prices. Moreover, the model is able to analyze the response of employment, unemployment and the labor force to exogenous shocks as well as finding the coefficients for an optimal simple rule.

Benigno and Ricci (2011) introduce DNWR in a DSGE model with flexible prices and find also that `greasing the wheels' and allowing for moderate inflation may help intratemporal and intertemporal relative wage adjustments and that those experiencing large volatility or lower productivity growth may find it desirable to target a higher inflation rate. They also link the steepness of the Phillips Curve and wage rigidities and find that the Phillips Curve would steepen if wage rigidities declined. Furthermore, when wage rigidities are present there exists a non-negligible long-run trade-off between inflation and the output gap, Benigno and Ricci (2011).

Schmitt-Grohé and Uribe (2016) motivated the manner in which the DNWR constraint is included as the constraint follows the same form utilised in their paper. Differences arise between our papers as Schmitt-Grohé and Uribe (2016) focus on

[4] See Kim and Ruge-Murcia (2009).

developing open economies and how the combination of a currency peg and free capital mobility creates a negative externality that causes overborrowing during booms and high unemployment during contractions. The value of the degree of DNWR, γ, is explored by Schmitt-Grohé and Uribe (2016) and is around 1 for the developed and developing countries they analyze. This paper follows Schmitt-Grohé and Uribe (2016) by choosing γ close to one. The majority of this paper uses $\gamma = 0.9975$ such that wages decline up to 1% per year, when a different value of γ is used it will be clearly outlined.

Fahr and Smets (2010) combines the convex cost function of Kim and Ruge-Murcia (2009) with regard to prices and wages with the labor market of Erceg et al. (2000). Using a two country model and real wage rigidity, instead of nominal rigidity, allows them to focus on transmission of monetary policy in a monetary union. Their main findings that pertain to this paper are that

> Downward nominal wage rigidities lead to a positively skewed response in nominal wage changes and a sizeable positive optimal inflation rate. This effect is stronger the lower price rigidity. The greasing effects of inflation vanish if wages are either indexed (real wage rigidity) or if adjustment costs are symmetric. (Fahr and Smets, 2010)

The range of optimal trend inflation proposed can be vast, with Gross (2018) finding optimal inflation in their model, which is an extension of Daly and Hobijn (2014), to be 5.4%. Gross (2018) uses a Calvo (1983) approach to the DNWR constraint, such that with some probability the household cannot lower their wage.

Moreover as well as adding motivation for positive trend inflation DNWR has also been shown to provide wage restraint. This has been shown in Wolf (2018), which also takes seriously the wage constraint found in Schmitt-Grohé and Uribe (2016) and uses it to assess wage inflation rates in the euro area.

More recently Dupraz et al. (2019)'s `A Plucking Model of Business Cycles' has brought further attention to the DNWR literature utilising the constraint found in Schmitt-Grohé and Uribe (2016). They embed a strict DNWR constraint into a search and matching framework to match the dynamics of U.S. unemployment. They find large benefits of increasing the inflation target, whereby the benefits found in this paper are modest. A main difference is the lack of price rigidities as well as the inclusion of a search framework, which is the source of our differences.

Thus this paper provides further support to grease the wheels of the economy and wage restraint. Moreover, the paper advances the literature by highlighting the differences in responses when the households follow a wage setting rule versus when they are forward looking and can optimise over the DNWR constraint. Furthermore, I find an optimal simple rule, one which can be followed by a central bank, which produces impulse responses from shocks close to the optimal monetary policy.

4 Methodology

The model is a fairly simple New Keynesian model but with unemployment as in Galí et al. (2011) and the addition of downward nominal wage rigidities. The model follows closely to a standard New Keynesian model that can be found in Galí (2015) `Chapter 6: Sticky Wages and Prices' as well as `Chapter 7: Unemployment and Monetary Policy'. The main difference between these models is that the one used in this paper suffers from Downward Nominal Wage Rigidities instead of Calvo wages.

4.1 Firms

As in the standard New Keynesian model, a continuum of firms are assumed to exist and are indexed by $i \in [0,1]$. Each firm produces a differentiated good with a technology represented by the production function

$$Y_t(i) = A_t N_t(i)^{1} \tag{1}$$

where A_t is an exogenous technology[5] parameter that is common to all firms, $Y_t(i)$ denotes the output of good i, and $N_t(i)$ is a labour input used by firm i and can

[5] Positive trend growth in technology is neglected here for simplicity, providing motivation for future extensions. Adding in positive growth in technology has important implications for optimal steady state price inflation.

thought of as employment or hours worked. The definition of $N_t(i)$ is given by

$$N_t(i) = \left(\int_0^1 N_t(i;j)^{1-\frac{1}{\epsilon_w}} dj \right)^{\frac{\epsilon_w}{\epsilon_w - 1}} : \tag{2}$$

Here $N_t(i;j)$ denotes the quantity of type-j labour employed by firm i in period t. Moreover, there is a continuum of labour types indexed by $j \in [0;1]$. The parameter ϵ_w represents the elasticity of substitution among labour types.

$W_t(j)$ denotes the nominal wage for type-j labour that prevails in period t, for all $j \in [0;1]$. Wages are set by the household and can be increased flexibly, however wages face downward rigidities such that the nominal wage cannot decrease freely - this is outlined further in Section 4.3.[6] Therefore the cost minimization yields a set of demand schedules for each firm i and labour type j, given the firm's total employment $N_t(i)$, which highlights that hiring of a particular labor type is due to their relative wage and substitutability

$$N_t(i;j) = \left(\frac{W_t(j)}{W_t} \right)^{-\epsilon_w} N_t(i) \tag{3}$$

for all $i;j \in [0;1]$, where

$$W_t \equiv \left(\int_0^1 W_t(j)^{1-\epsilon_w} dj \right)^{\frac{1}{1-\epsilon_w}} \tag{4}$$

is an aggregate wage index. Through substituting equation (4) into equation (3) it can be shown that

$$\int_0^1 W_t(j) N_t(i;j) dj = W_t N_t(i);$$

which is a convenient aggregation result that will subsequently be used.

As well as hiring workers firms set the price of final goods in the economy following Calvo (1983), which is typical in a New Keynesian model. A firm in period t will choose the price P_t^* to maximize their current market value of profits,

[6] The assumption of upwardly flexible wages is further explored in Appendix D.3 as Calvo wage rigidity is introduced as in Galí (2015) and different probabilities for labour unions to change their wage is explored in Figure 10.

however, a firm may only reset their price with a probability $1-\theta_p$ in any given period. Hence their problem can be shown to be,

$$\max_{P_t^*} \sum_{k=0}^{\infty} \theta_p^k E_t\left\{ \Lambda_{t,t+k}(1/P_{t+k}) \left(P_t^* Y_{t+k|t} - \mathcal{C}_{t+k}(Y_{t+k|t}) \right) \right\}$$

subject to the sequence of demand constraints

$$Y_{t+k|t} = \left(\frac{P_t^*}{P_{t+k}} \right)^{-\epsilon_p} C_{t+k} \tag{5}$$

for $k = 0, 1, 2, \ldots$ where $\Lambda_{t,t+k} \equiv \beta^k U_{c,t+k}/U_{c,t}$ is the stochastic discount factor, $\mathcal{C}(\cdot)$ is the nominal cost function. As shown in Galí (2015) solving this problem and rearranging accordingly, as well as conducting a first-order taylor-approximation in the neighborhood of the zero inflation steady state, the following equation for price inflation $\pi_t^p \equiv p_t - p_{t-1}$[7]

$$\pi_t^p = \beta E_t\{\pi_{t+1}^p\} - \lambda_p \hat{\mu}_t^p \tag{6}$$

where as in Galí (2015), $\hat{\mu}_t^p \equiv \mu_t^p - \mu^p$ is the deviation of the average (log) price markup from its flexible price counterpart and $\lambda_p \equiv \frac{(1-\theta_p)(1-\beta\theta_p)}{\theta_p} \frac{1-\alpha}{1-\alpha+\alpha\epsilon_p}$. Firms wish to raise their prices when the average price markup in the economy today or in the future is below the desired levels of the firms and hence prices rise when firms are able to change their prices and inflation arises from this. The average price markup is related to the output and real wage gaps, whereby using $\hat{\mu}_t^p = mpn_t - \omega_t$, one can write the New Keynesian Phillips Curve as:

$$\pi_t^p = \beta E_t\{\pi_{t+1}^p\} + \kappa_p \tilde{y}_t + \lambda_p \tilde{\omega}_t$$

with $\kappa_p \equiv \frac{\alpha\lambda_p}{1-\alpha}$.

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[7] Lower case letters throughout this short paper denote the (natural) log of the corresponding variable, e.g., $p_t \equiv \log P_t$

4.2 Households and Unemployment

This section follows closely Galí (2015) Chapter 7 on unemployment and monetary policy.

4.2.1 Households

There exists a large number of identical households, whereby each household has a continuum of members represented by the unit square and indexed by a pair $(j, s) \in [0,1] \times [0,1]$. Here $j \in [0,1]$ represents the type of labour that the household member specialises in and $s \in [0,1]$ is the disutility that each household member faces from working. Disutility from work is given by χs^{φ} if he is employed and zero otherwise, where $\chi > 0$ and $\varphi > 0$ are exogenous parameters. Full risk sharing within the household is assumed and therefore given the separability of preferences this implies the same level of consumption for each household member. The household's period utility is given by the integral of its members' utilities and can therefore be written as follows

$$
\begin{aligned}
U(C_t, \{N_t(j)\}; Z_t) &\equiv \left(\frac{C_t^{1-\sigma} - 1}{1-\sigma} - \chi \int_0^1 \int_0^{N_t(j)} s^{\varphi} \, ds \, dj \right) Z_t \\
&= \left(\frac{C_t^{1-\sigma} - 1}{1-\sigma} - \chi \int_0^1 \frac{[N_t(j)]^{1+\varphi}}{1+\varphi} dj \right) Z_t
\end{aligned}
$$

where $C_t \equiv \left(\int_0^1 C_t(i)^{1-\frac{1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}$ is a consumption index, $C_t(i)$ is the quantity consumed of good i, for $i \in [0,1]$, and $N_t(j) \in [0,1]$ is the fraction of members specialised in type j labour who are employed in period t. The preference shifter, z_t, is assumed to follow an AR(1) process with $\rho_z = 0.5$ and ε_t^z is a white noise process with zero mean and variance $\sigma_z^2 = 1$ and can be rationalised as a demand shock.

$$
z_t = \rho_z z_{t-1} + \varepsilon_t^z
$$

Each household seeks to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, \{N_t(j)\}; Z_t)$$

subject to a sequence of flow budget constraints given by

$$\int_0^1 P_t(i)C_t(i)di + Q_t B_t \leq B_{t-1} + \int_0^1 W_t(j)N_t(j)dj + D_t. \tag{7}$$

Here $P_t(i)$ is the price of good i, $W_t(j)$ is the nominal wage for labour type j and B_t represents purchases of a nominally riskless one-period bond, Q_t is the price of that bond and D_t is a lump-sum component of income, which can be thought of as dividends from ownership of firms.

Optimal demand for each good resulting from utility maximization takes the form:

$$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon_p} C_t \tag{8}$$

where $P_t \equiv \left(\int_0^1 P_t(i)^{1-\frac{1}{\epsilon_p}} di\right)^{\frac{\epsilon_p}{\epsilon_p-1}}$ denotes the price index for final goods. This takes a familiar form and can be shown that $\int_0^1 P_t(i)C_t(i)di = P_t C_t$.

The household's intertemporal optimality condition is given by and Euler equation of the form

$$Q_t = \beta E_t \left\{ \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \left(\frac{Z_{t+1}}{Z_t}\right) \left(\frac{P_t}{P_{t+1}}\right) \right\} \tag{9}$$

The wage setting is done by the workers, or a union that represents all workers specialised in it. The innovation over a standard New-Keynesian model is to have wages adhere to DNWR as seen in Section 4.3.

4.2.2 Unemployment

Unemployment in this model follows that of Galí et al. (2011), hence unemployment arises due to the discrepancy in wages set by the labour union and firm. Taking into account the household members' disutility from working that individual will be willing to work, and therefore be a part of the labour force if and only if

$$\frac{W_t(j)}{P_t} \quad C_t s^{\varphi}:$$

Therefore, the individual will be willing to work if the real wage achieved exceeds the disutility from working given in units of consumption, hence multiplied by the household's marginal utility of consumption.

The marginal supplier of type j labour, denoted $L_t(j)$, is given by

$$\frac{W_t(j)}{P_t} = \quad C_t L_t(j)^{\varphi} \tag{10}$$

Following this we can de ne the aggregate labour force by integrating over labour types j, such that $L_t \quad \int_0^1 L_t(j)dj$. Taking logs and integrating over j provides the following approximate relation for the real wage :

$$w_t \quad p_t = \quad c_t + \varphi l_t + \quad : \tag{11}$$

Equation (11) can be thought of as a participation equation where by rst-order approximation around the symmetric steady state $w_t \simeq \int_0^1 w_t(j)dj$, $l_t \simeq \int_0^1 l_t(j)dj$ and $\log(\;) = \;$.

Following Galí (2011); Galí et al. (2011) the unemployment rate u_t is de ned as the log di erence between the labour force and employment:

$$u_t \quad l_t \quad n_t: \tag{12}$$

Combining the average wage markup $^{w}_{t} \quad (w_t \quad p_t) \quad (\;c_t + \varphi n_t + \;)$ with equation (11) and equation (12), provides us with a linear relation between the wage markup and the unemployment rate

$$^{w}_{t} = \varphi u_t: \tag{13}$$

Employment is demand determined with the labour demand given by the inverse production function in logs

$$n_t = \frac{1}{1 \quad}(y_t \quad a_t): \tag{14}$$

Following Galí (2015) the natural rate of unemployment, u^n_t, is de ned as that which would prevail in the absence of nominal wage rigidities. The natural rate of unemployment is set to 0.05, consistent with an average unemployment rate of 5%, to take into account frictional unemployment.

$$u^n = \frac{\mu^w}{\varphi}: \tag{15}$$

It is important to note that due to the monopoly the households have over labour that even under flexible wages a wage markup, μ^w, will still be positive and hence natural rate of unemployment will also be greater than 0.

4.3 Wage Setting and Downward Nominal Wage Rigidity

Households set wages by maximising their utility with respect to their budget constraint as well as the sequence of labour demand schedules given in equation (3). Since the households have market power over wage setting in a flexible wage setting environment, one without any nominal rigidities, they would set wages as in equation (16) as a markup over their marginal rate of substitution. The markup here is given by as $M_w \equiv \frac{\epsilon_w}{\epsilon_w - 1}$.

$$\frac{W_t}{P_t} = \frac{W_t}{P_t} = M_w C_t^{\sigma} N_t^{\varphi} \tag{16}$$

The main innovation of this paper is the inclusion of Downward Nominal Wage Rigidities that has been inspired by Schmitt-Grohé and Uribe (2016) and empirically motivated in Section 2. With DNWR the occasionally binding constraint is imposed of

$$W_t \geq \gamma W_{t-1} \quad \gamma > 0; \tag{17}$$

where γ defines the degree of downward nominal wage rigidity. Such that when $\gamma = 0$ there is full wage flexibility and the higher γ, the more downwardly rigid are nominal wages. If $\gamma \geq 1$ we see absolute downward wage rigidity, found empirically in Schmitt-Grohé and Uribe (2016) for many countries[8]. The parameter γ is chosen to emulate that of Schmitt-Grohé and Uribe (2016), such that $\gamma = 0.9975$,

[8] See Schmitt-Grohé and Uribe (2016) for an extensive list, as an example it includes countries such as Bulgaria, Ireland, Italy, Spain, Slovenia.

which lies within the estimated bounds found, and at a quarterly frequency implies that nominal wages can decline up to 1 percent per year.[9] Therefore the wage setting rule divided by the price level for convenience can now be written as:

$$\frac{W_t}{P_t} = \max\left\{ \mathcal{M}_w C_t^{\sigma} N_t^{\varphi} \; ; \; \gamma W_{t-1} \frac{1}{P_t} \right\} \tag{18}$$

For the first half of this paper the downward nominal wage rigidity constraint is added into the model exogenously. This means that the households are only able to see the constraint once they reach it. This lends itself to a first attempt at understanding the effect of adding in such an occasionally binding constraint and provides a simple modelling environment using Occbin as discussed in Guerrieri and Iacoviello (2015) and explained in Appendix B for this model.

Later in section 6 households will be able to maximise their utility with respect to their wage while taking into account the downward nominal wage constraint. A model of this type cannot be solved using pertubation techniques and therefore I use Smolyak collocation, a projection method, to solve the model. The computational technique is outlined in Appendix C.2.

4.4 Equilibrium and Calibration

Below are the equations that characterize the equilibrium conditions in the New Keynesian Framework developed above. It is important to note that these correspond exactly to that in Galí (2015) except equation (21), which under Calvo wages would display an equation relating wage inflation to output and the real wage gap. Due to the assumption of flexible wages and the DNWR constraint equation (21) shows instead how real wages are set as a markup over the marginal rate of substitution, unless the downward nominal wage rigidity is binding. Where $\tilde{y}_t = y_t - y^n$, the output gap and $\tilde{\omega}_t$ is the real wage gap.

$$\tilde{y}_t = -\frac{1}{\sigma}\left(i_t - E_t\{\pi^p_{t+1}\} - r^n_t\right) + E_t\{\tilde{y}_{t+1}\} \tag{19}$$

$$\pi^p_t = \beta E_t\{\pi^p_{t+1}\} + \kappa_p \tilde{y}_t + \lambda_p \tilde{\omega}_t \tag{20}$$

[9] Figure 9 which can be found in Appendix D.2 highlights the impact of different γ on the economy and conclusions. A lower value of γ moves the economy closer to that of flexible wages.

$$\omega_t = \max\{ \ ^{w} + \quad + c_t + \varphi n_t ; \quad + \omega_{t \ 1} \quad ^{p}_{t}\} \tag{21}$$

$$\vdash_t \quad \vdash_{t \ 1} + \ ^{w}_{t} \quad ^{p}_{t} \quad \omega^{n}_{t} \tag{22}$$

$$\text{Simple Rule 1 (Taylor):} \qquad i_t = \quad + \quad ^{p}_{t} + \ _{y} \tilde{y}_t \tag{23}$$

The calibration used is standard except the downward nominal wage rigidity parameter , which is taken from Schmitt-Grohé and Uribe (2016).

Table 1: Table summarizing parameter values

Symbol	Parameter	Value	Reasoning
	Capital Share	0.25	Standard
	Discount Factor	0.99	Standard
'	Inverse Frisch elasticity	5	Galí (2015)
p	Demand elasticity for goods	9	Galí (2015)
w	Demand elasticity for labour services	4.5	Galí (2015)
p	Calvo parameter for price	0.75	Galí (2015)
	Taylor weight on in ation	1.5	Galí (2015)
y	Taylor weight on output gap	0.125	Galí (2015)
u^n	Natural rate of unemployment	5%	Galí (2015)
	DNWR Parameter	[0.99,0.9975]	4% to 1% wage de ation

5 Results

This section houses the main results of the paper for the simple model, whereby the model is solved around a zero in ation steady state and exogenous downward nominal wage rigidity constraint. A more realistic model that is solved around positive trend in ation and allows the household to maximise over the occasionally binding constraint can be seen in Section 6.

The gures below highlight how a simple New-Keynesian economy is a ected by adding in downward nominal wage rigidities. Of signi cance is the asymmetric response of the interest rate under monetary policy rules, as well as the optimal monetary policy. Furthermore, comparing the welfare loss under di erent monetary policy rules motivates exploration for an optimal simple rule that attempts to mimic the optimal monetary policy.

Figure 1 contrasts the response of a New-Keynesian model with exible wage versus downward nominal wage rigidities. As shown below, under exible wages the wage is free to adjust such that nominal wages initially rise and then fall faster than when they are constrained by the downward nominal wage rigidity. The prolonged higher wage, when wages are constrained, pushes up prices and causes the central bank to raise the nominal interest rate, which causes households to save rather than consume. This lack of demand causes employment to fall.[10] Following this positive demand shock, due to the increased wage even when the shock subsides we see an increase in unemployment as seen in Schmitt-Grohé and Uribe (2016) as employment falls, through the change in aggregate demand resulting from the endogenous monetary policy response, but labour supply increases due to the higher wage. This results subsides when the downward nominal wage rigidity constraint is internalised in Section 6. The persistence in the nominal wage causes a boom-bust cycle as seen in the output gap in Figure 1. Since this model does not feature capital the movements in output are driven entirely by technology, A_t, and employment, N_t. The high price in ation caused by the arti cially high wages helps to de ate the real wage back to its steady state value.

Figure 1: Positive Demand Shock under the Simple Rule 1 (Taylor)

Impulse response for variables facing a positive demand shock. The demand shock follows an AR(1) with $_z = 0{:}5$ and $= 1$. The Simple Rule 1 (Taylor) is $i_t = 0{:}01 + 1{:}5\ ^p_t + 0{:}125\hat{y}_t + \ _t$.

[10] For a clear exposition of the determinant of employment in the New Keynesian model see Galí (2013).

Figure 2 highlights the asymmetric response of the economy due to the inclusion of the Downward Nominal Wage Rigidity. From a positive shock, as seen in greater detail in Figure 1 a boom-bust cycle appears in the output gap. However, from a negative demand shock since nominal wages are unable to fall we see an amplification of the fall in output, which is caused by a greater fall in employment compared to the flexible wage counterpart. When wages are assumed to be flexible, or are afflicted by wage rigidity in the form of Calvo (1983), the outcome on the output gap is symmetric. This symmetry is broken in this model due to the inclusion of the occasionally binding constraint and thus we see asymmetric response of the economy to symmetric shocks. The full response of the economy due to a negative demand shock can be seen in Figure 8 of Appendix D.1.

Figure 2: Asymmetric Output Gap Movement from symmetric Demand Shock

Output gap from a positive or negative symmetric demand shock. The demand shock follows an AR(1) with $_z = 0.5$ and $= 1$. The Simple Rule 1 (Taylor) is $i_t = 0.01 + 1.5\,^{p}_{t} + 0.125\hat{y}_t + \,_t$.

Due to the constraint occasionally binding we see in Figure 3 the asymmetric response of monetary policy when the central bank follows a simple rule in the form of the standard Taylor rule. Under flexible or Calvo wages the response of the central bank is symmetric, however, when the economy suffers from DNWR the central bank reaction is stronger under a positive demand shock than a negative demand shock. This is because under a positive demand shock nominal wages rise, pushing up in ation and causing the central bank to react strongly, however, since wages cannot fall during a negative demand shock in ation will remain close to its steady state where the central bank, which cares most about in ation deviations, will not need to react as strongly as before to guide in ation back to its steady state.

Figure 3: Asymmetric Annual Interest Rate Response to a Symmetric Demand Shock

Impulse response of the nominal interest rate facing a positive and negative demand shock. The demand shock follows an AR(1) with $_z = 0{:}5$ and $= 1$. The Simple Rule 1 (Taylor) is $i_t = 0{:}01 + 1{:}5\ {}^p_t + 0{:}125\hat{y}_t + {}_t$.

5.1 Optimal Monetary Policy

The optimal monetary policy is a perfect-foresight solution derived from minimising the discounted sum of welfare loss, shown in equation (24), subject to the equilibrium condition in section 4.4. The exogenous introduction of DNWR does not alter the steady state of the model nor the second-order approximation to the utility of the representative consumer, which remains the standard derivation as shown in Galí (2015) for a New Keynesian model with price rigidities. The introduction of the DNWR, similar to the introduction of the zero-lower-bound, restricts the set of feasible equilibrium paths such that whilst the constraint binds the optimal allocation, characterised by zero inflation and zero output gap at all times, cannot be attained. However, as is standard in the New Keynesian model the optimal monetary policy is able to fully neutralise any effect a demand shock would have on the welfare loss through changes in the interest rate. This is not the case when faced with a technology shock as it affects the natural output and interest rate of the economy. Without the DNWR constraint the optimal policy following a technology shock can replicate the flexible price/flexible wage equilibrium allocation through the necessary adjustments of the real wage (setting the real wage equal to the natural real wage at every point in time t). The inclusion of the DNWR breaks this relationship as wages are no able to decrease flexibly, and therefore cannot adjust adequately. Even with the inclusion of the DNWR constraint the welfare loss function used is the same as in the Calvo price but flexible wage New Keynesian model since the inclusion of this occasionally binding constraint does not affect the efficient steady state nor does it cause any wage dispersion. Therefore, as in Amano and Gnocchi (2017), which features an analogous DNWR constraint, the second-order Taylor expansion of utility is well defined because the expressions needed to derive it, the utility function and feasibility constraints, are differentiable whilst the inequality constraint is not needed in the approximation. Hence, the welfare loss function is:

$$W = \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left[\left(\sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) \tilde{y}_t^2 + \frac{\epsilon_p}{\lambda_p} (\pi_t^p)^2 \right] \tag{24}$$

Moreover, the average welfare loss used to compare different monetary policy rules is derived from the standard Calvo price but flexible wage New Keynesian model and is given by:

$$L = \frac{1}{2} \left[\left(\sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) var(\tilde{y}_t) + \frac{\epsilon_p}{\lambda_p} var(\pi_t^p) \right] \tag{25}$$

The central bank's optimal monetary policy problem is outlined and solved under the more general model assumption of Calvo price and wage rigidities as well as the DNWR constraint in Appendix A.

The optimal monetary policy response to a symmetric (positive or negative) demand shock is symmetric, due to the aforementioned reasons. Therefore figure 4 analyses if the optimal monetary policy is asymmetric following a positive and negative technology shock of one standard deviation and three standard deviations. Figure 4 shows that it is an optimal response of the central bank to act asymmetrically. This asymmetry arises due to the occasionally binding constraint and the central bank reacting to minimise movement in the inflation and the output gap. The kinked response of the central bank under the optimal monetary policy is amplified as the size of the shock increases, due to nominal wages stuck at the DNWR constraint for longer. A positive technology shock in the New Keynesian model with standard parameter values and flexible wages leads to a fall in inflation and employment as well as a decline in nominal wages. With DNWR and under a Taylor Rule, a similar picture emerges, albeit with nominal wages sluggishly falling to their new steady-state level. The optimal monetary policy adjusts the interest rate such that wages remain high, counteracting the relative reduction in cost of output for the firm, and price inflation stays at the steady-state. Following a negative technology shock the central bank acts to reduce fluctuations in price inflation dampening any nominal wage increases that would typically accompany this shock.

Figure 4: Asymmetric Optimal Monetary Policy: Technology Shock

Impulse response of the nominal interest rate under optimal monetary policy facing a positive and negative technology shock. The technology shock follows an AR(1) with $_a = 0.9$ and $= \{1; 3\}$.

5.2 Simple Rule 2

Figure 5 assesses whether current monetary policy rules can match the optimal monetary policy response in this environment. To compare against Simple Rule 1 (Taylor) another simple rule is used, which is taken from Galí (2015) and performs well under Calvo price and wage rigidities. $\hat{u}_t$ here is defined as the log difference between the unemployment rate and natural level of unemployment:

$$\text{Simple Rule 2 (Galí):} \qquad i_t = 0.01 + 1.5\pi^p_t - 0.5\hat{u}_t. \tag{26}$$

In Figure 5 it can be seen that Simple Rule 2 (Galí) proposed in Galí (2015) performs well in limiting the change in the output gap due to reacting to changes in the unemployment, however the response of price inflation is closer to the Taylor rule than the optimal monetary policy. After a positive technology shock the optimal monetary policy allows the nominal wage to rise, which adds pressure on price in ation and keeps it at its steady state level. In this scenario the positive technology shock helps to counteract the higher nominal wages to keep employment and therefore the output gap at their steady state levels.

In contrast, the optimal response, the response which minimises the welfare loss, allows the wage to fall (shown as an increase in the Figure as the impulse response has been multiplied by -1) and minimises any movement in price in ation and the output gap from a negative productivity shock. The fall in the real wage may seem counter intuitive but due to a fall in aggregate productivity, A_t, the marginal cost of production increases and therefore allowing wages to fall helps to o set this increased cost of production. Simple Rule 2, which is the simple rule of Galí (2015), where the central bank reacts to in ation and unemployment, outperforms the standard Taylor rule (Simple Rule 1) and does not react asymmetrically to the productivity shocks as nominal wages do not hit the occasionally binding constraint.

Figure 5 highlights the asymmetric response to a symmetric technology shock, as can be seen in the movements in the annual interest rate under the optimal monetary policy and the Taylor Rule.

Figure 5: Optimal Policy vs Simple Rule 1 (Taylor) vs Simple Rule 2 (Galí): Positive and Negative Technology Shocks

Impulse response for variables facing a positive and negative technology shocks. The technology shocks follows an AR(1) with $_a = 0.9$ and $= 1$. Simple Rule 1 (Taylor) is $i_t = 0.01 + 1.5\,{}^p_t + 0.125\hat{y}_t + {}_t$ and Simple Rule 2 (Galí) is $i_t = 0.01 + 1.5\,{}^p_t \quad 0.5\hat{a}_t$. The impulse responses for the negative technology shock have been multiplied by -1.

5.3 Simple Rule 3

The relatively good performance of the Simple Rule 2 (Galí) in Figure 5 and welfare losses presented in Table 2 and Table 5 in Appendix D.4 compared with the Simple Rule 1 (Taylor) motivated a search to see if another simple rule could minimise the welfare loss. The coe cients on the new optimal simple rule are found by simulating the economy over demand and technology shocks and optimising these values to produce the minimise the welfare loss. The starting point of this search is a general rule of the form:

$$i_t = 0.01 + {}_i i_{t\ 1} + {}^p_t + {}_y\hat{y}_t + {}_w\,{}^w_t + {}_a\hat{a}_t + {}_t; \tag{27}$$

where dictates the sensitivity of the nominal interest rate to each corresponding variable. Simple Rule 3, the optimal simple rule displayed in equation (28) is similar to the simple rule provided in Galí (2015), however, reacts stronger to both inflation and the unemployment gap - difference in unemployment and its natural rate. Moreover, this new optimal simple rule assigns a higher weight on deviations in unemployment compared compared to inflation than Simple Rule 2 (Galí) in equation (26).[11] [12] Here $\rho = -\log(\beta) = 0.01$ and therefore the OSR can be given as:

$$\text{Simple Rule 3 (Optimal Simple Rule)} \quad : \quad i_t = 0.01 + 4.0\pi_t^p - 2.50\hat{u}_t. \quad (28)$$

Figure 6 presents the impulse response from positive and negative technology shocks comparing the optimal monetary policy, optimal simple rule and the simple rule. The optimal simple rule closely follows the optimal monetary policy and is able to approximately replicate the optimal monetary policy. In comparison to Figure 5, which shows the outcome under a Taylor rule, output gap deviations have been significantly dampened. Paradoxically the optimal monetary policy and optimal simple rule allow for increases in nominal wage from a positive technology shock even though wage decreases are sluggish. Allowing a higher wage allows for price inflation to fall less and the labour force decrease to be muted. Under the optimal simple rule the output gap also deviates less compared to the alternative simple rule, due to employment staying closer to its steady state value. Simple Rule 3, the optimal simple rule, exhibits asymmetry following symmetric technology shocks, with a similar motive to the optimal monetary policy - adjusting to the interest rate to allow for nominal wages to fall sufficiently to counteract the lower aggregate productivity from a negative technology shock. Unlike the optimal monetary policy, which also benefits from the assumption that the central bank acts under commitment, the response following a positive and negative technology shock only deviate under

[11] Debortoli et al. (2019) also find welfare improvements through imposing a higher weight on the unemployment gap or the output gap than the standard Taylor rule.

[12] Finding the optimal simple rule coefficients numerically has the drawback that additional welfare loss minimisation can still be achieved. However, the only improvement to this rule found so far requires very large coefficients on the simple rule for minimal gains.

Simple Rule 3 (OSR) for two quarters.

Figure 6: Optimal Monetary Policy, Simple Rule 3 (OSR), Simple Rule 2 (Galí): Positive and Negative Technology Shocks

Impulse response for variables facing a positive and negative technology shocks. The technology shocks follows an AR(1) with $_a = 0{:}9$ and $= 1$. Simple Rule 2 (Galí) is $i_t = 0{:}01 + 1{:}5\ {}^{p}_{t}\quad 0{:}5\tilde{y}_t$ and Simple Rule 3 (OSR) is $i_t = 0{:}01 + 4{:}0\ {}^{p}_{t}\quad 2{:}5\tilde{y}_t$. The impulse responses for the negative technology shock have been multiplied by -1.

Table 2 displays the welfare loss, using equation (25), from positive technology and demand shocks. The negative shock counterpart to this Table can be found in Appendix D.4 Table 5. Strict targeting rules keep price inflation and wage inflation, respectively, at their steady state values and adjust the interest rate accordingly. The optimal rule provides a lower bound on the welfare loss in the Table. From Table 2 it is evident that Simple Rule 3 (OSR) performs well with both positive technology and demand shocks. This is in contrast to Simple Rule 1 (Taylor), which provides a relatively high welfare loss in comparison to the other rules in Table 2.

Table 2: Evaluation of MP rules following positive Technology and Demand Shocks

	Optimal	Strict Targeting		Simple Rules 1, 2, & 3		
		Price	Wage	Taylor	Galí	OSR
Technology shocks						
$(\ ^{p})$	0	0	0.025	0.076	0.042	0.009
$(\ ^{w})$	0.1905	0.1905	0	0.090	0.054	0.158
$(\tilde{y})$	0	0	0.0334	0.664	0.026	0.06
L	0	0	0.133	1.232	0.364	0.016
Demand Shocks						
$(\ ^{p})$	0	0	0	0.059	0.111	0.030
$(\ ^{w})$	0	0	0	0.551	0.824	0.367
$(\tilde{y})$	0	0	0	0.067	0.114	0.054
L	0	0	0	0.756	2.6501	0.206

Simple Rule 1 (Taylor) is the Taylor rule with $i_t = 0{:}01 + 1{:}5 \ ^{p}_{t} + 0{:}125\tilde{y}_t + \ _t$ and Simple Rule 2 (Galí) is $i_t = 0{:}01 + 1{:}5 \ ^{p}_{t} \ \ 0{:}5\tilde{u}_t$. Simple Rule 3 (OSR) is $i_t = 0{:}01 + 4{:}0 \ ^{p}_{t} \ \ 2{:}5\tilde{u}_t$.

6 Extensions

This section houses part of the extensions of the model presented in the main text above. The main di erences are i) the model is not log-linearised around a zero percent steady-state in ation rate and ii) the model internalises the occasionally binding constraint, which allows the households to maximise their utility taking into account that wages are downwardly rigid. This moves the model to a more appealing setting, enables the exploration of the optimal trend in ation rate and provides more sensible results to exogenous shocks.[13] The results in this section are presented with Simple Rule 1 (Taylor) assumed.

Internalising the downward norminal wage rigidity constraint means that the household's maximisation problem needs to be revisited. Since the only optimisation problem impacted is the wage maximisation this is the focus on the equations below. The variables in parenthesis $_t$ and $_t$ correspond to the lagrange multipliers, or shadow cost of the constraints. The wage setter (household or labour union for a worker of type j) seeks to maximise their utility ow subject to labour demand, the budget constraint and the downward nominal

[13] The main illustration of this is the fall in employment from a positive demand shock seen in Figure 1.

wage rigidity.

$$\max_{W_t(j)} E_0 \left\{ \sum_{t=0}^{1} {}_{t} \left[\frac{C_t^{1}}{1} \quad \frac{N_t(j)^{1+}}{1+} \right] Z_t \right\}$$

subject to:

$$N_t(j) = \left(\frac{W_t(j)}{W_t} \right)^{w} N_t$$

$$(\; _t) \quad P_tC_t + E_t[Q_tD_{t+1}] \quad D_t + W_t(j)N_t(j) \quad T_t$$

$$(\; _t) \quad W_t(j) \quad W_{t \; 1}(j)$$

The solution to this problem, combining the previous rst order conditions found in Section 4, can be seen below. It is convenient when simulating the model to represent this condition in terms of nominal wage in ation $^{w}_{t} = W_t/W_{t \; 1}$ and the real wage rather than solely nominal wages and nominal wage changes.

$$^{w}_{t} \quad _t = (\; _w \quad 1)\frac{W_t}{P_t} Z_t C_t \quad N_t \quad _w N_t^{1+} \; Z_t + \; E_t[\; _{t+1} \quad ^{w}_{t+1}]$$

$$\text{Complementary slackness: } _t(\; ^{w}_{t} \quad) = 0$$

Non-negativity constraints can be shown to be:

$$_t \quad 0$$

$$^{w}_{t} \quad 0$$

Therefore when the DNWR constraint does not bind the associated lagrange multiplier will equal zero, $_t = 0$, and we are back to the exible wage schedule where the real wage is a markup over the marginal rate of substitution between consumption and labour. A detailed derivation of this problem can be found in Appendix B.1.

6.1 Extension Results

Figure 7 displays the impulse response from a one standard deviation positive demand shock, comparing the response of an economy with downward nominal wage rigidities and flexible wages under a Taylor rule with $\phi_\pi = 1.5$ and $\phi_y = 0.125$. For this figure no positive trend inflation is assumed, which allows for a direct comparison to Figure 1. Figure 7 can be used as a comparison to Figure 1 which had exogenous DNWR and a zero inflation steady state. Internalising the occasionally binding constraint means that the households now barely increase their nominal wage, which is in contract to the increase of of 2% witnessed previously. Therefore the existence of the constraint causes wage increases to be muted, creating an endogenous rigidity when increasing wages. Figure 7 also shows a more sensible response of the labour market to a positive demand shock since the labour force response is muted (not shown) due to the subdued wage, whilst employment still rises, hence unemployment falls on impact of the shock and output increases similarly with the flexible wage model.

Figure 7: Taylor Rule - Positive Demand Shock

Impulse response for variables facing a positive demand shock using projection methods. The demand shock follows an AR(1) with $_z = 0:5$ and $= 1$. The Taylor rule is $i_t = 0:01 + 1:5 \, {}^{p}_{t} + 0:125\hat{y}_t + {}_t$. Wages are assumed to be either exible or su er from downward nominal wage rigidities that are internalised by the labour union. The DNWR parameter is $= 0:9975$.

Wage restraint, the phenomenon displayed in Figure 7, has also been found empirically in Elsby (2009) and motivated by a stylised model of workers resistant to nominal wage cuts. Instead in this paper workers understand that wages are downwardly rigid and therefore limit their demand for higher wages as unemployment will arise when the DNWR constraint binds. This mechanism is due to the equilibrium wage being arti cially high if the DNWR constraint binds, which causes labour supply to remain high, the wage to not adjust downwards and therefore the rm cannot a ord to hire all available workers and unemployment arises. This form of wage restraint is similar to the benchmark case of including Calvo wage rigidity into a model with downward nominal wage rigidities as seen in Appendix D.2 and speci cally Figure 10.

6.2 Optimal Trend Inflation and Taylor Rule

Unlike in Section 5, I now introduce welfare as the present discounted value of the flow utility of a representative agent, which will be used to assess the optimal steady state inflation rate and Taylor rule coefficients under DNWR. The previous measurement of optimality, which used a second order approximation around a zero inflation steady state, cannot be used to assess positive trend inflation in that form. Moreover, this measure should be able to handle the highly non-linear nature of the occasionally binding constraint and therefore provide a more accurate measure of welfare. The optimal inflation and Taylor rule coefficients will disciplined by choosing the values which maximise the present discounted value of the flow utility of a representative agent seen in equation (29).

$$V_t = U_t(C_t; N_t) + \beta E_t V_{t+1} \tag{29}$$

In contrast, the Ramsey Planner, which provides the optimal solution to this model, would maximise the households welfare taking into account the first order conditions from our non-linear model seen in Appendix B. For now I focus on a standard Taylor rule that focuses on deviations in inflation and output from their steady state levels, outlined below.

$$\frac{R_t}{R} = \left(\frac{\pi_t}{\pi}\right)^{\phi_\pi} \left(\frac{Y_t}{Y}\right)^{\phi_y} \tag{30}$$

Using a grid search method[14] over $\{\pi; \phi_\pi; \phi_y\}$ the economy is simulated for 300,000 periods of shocks[15] and the mean value of the households welfare is calculated V_t and transformed into its consumption equivalent amount in comparison to a zero trend inflation steady state and Taylor rule coefficients $\{\phi_\pi = 1{:}5; \phi_y = 0{:}25\}$. The finding is suggestive of `greasing the wheels' and is within a sensible range of what others have found. In this model the optimal trend inflation rate is dependent on shocks and is between 0% to 1.25% with

[14] Future work which will provide a robustness check will use a non-linear solver such as the Newton-Raphson Method to determine the optimal $\{\pi; \phi_\pi; \phi_y\}$. Moreover, technological growth will need to be added to the model as well as this plays an important role in finding the optimal level of trend inflation.

[15] Technology and Demand shocks are simulated separately and the welfare values from the simulation are then compared.

$\phi_\pi \in [2;5]$ and $\phi_y \in [0;0.25]$. The higher trend inflation helps to deflate the Downward Nominal Wage Rigidity with the higher-than-typical reaction to inflation likely being needed to assure determinacy of the model. Other papers, such as Kim and Ruge-Murcia (2009) who use asymmetric wage adjustment costs, find that the optimal trend inflation is 0.35%. However, using asymmetric wage adjustment costs and heterogeneous agents Fagan and Messina (2009) find a much larger range of optimal trend inflation, 0% to 5% depending on calibration used. The tables below summarise the optimal calibration exercise:

Welfare Analysis: Optimal calibration of Taylor Rule

Table 3: Demand shocks

$\bar{\pi}$	ϕ_π	ϕ_y	Consumption Equivalence
0.25%	2	0	0.64%
0%	2	0	0.63%
0%	1.5	0	0.62%

Table 4: Technology shocks

$\bar{\pi}$	ϕ_π	ϕ_y	Consumption Equivalence
1.00%	5	0.25	0.71%
1.00%	4.5	0.25	0.69%
1.25%	5	0.25	0.67%

Consumption equivalence is calculated from differences in the mean of discounted household flow utility ($V_t = U_t(C_t;N_t) + \beta E_t V_{t+1}$) after 300,000 periods of uniformly distributed shocks. The welfare compared is from a Taylor rule with $\{\bar{\pi} = 0\%;\ \phi_\pi = 1.5;\ \phi_y = 0.25\}$. DNWR parameter $\gamma = 0.9975$ allowing for an annual decrease of nominal wages by 1%.

Further work will be completed to extend the model, mimicking the work done above to find an optimal simple rule in this set-up - providing a further contribution to the literature.

7 Conclusion

In conclusion adding an occasionally binding constraint into the New-Keynesian model such as a downward nominal wage rigidity seen throughout this paper can distort the standard results of a New-Keynesian model with exible or Calvo wages. This work has found that a DNWR constraint can cause boom-bust cycles from a positive demand shock if the agents within the model are not a ected by the constraint until they receive a shock that will cause them to reach the constraint. The optimal monetary policy in this setup is asymmetric and there are gains in welfare to be made over the Taylor rule by nding a new optimal simple rule - one that reacts stronger to changes in unemployment. Taking the constraint seriously and embedding it into the households problem and solving the non-linear model with positive trend in ation leads to support for `greasing the wheels', allowing positive in ation in the steady state to de ate real wage changes, which leads to welfare gains. Once the constraint is fully internalised, such that the wage setters understand its existence even during periods that they are unconstrained, wage increases become dampened even though they are exible upwards, a nding also shown in Elsby (2009) and Wolf (2018). The main contribution of this work comes from embedding the DNWR constraint from Schmitt-Grohé and Uribe (2016) into a New-Keynesian model, nding wage restraint and a new optimal simple rule whilst providing more support to positive trend in ation.

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A Derivation of Optimal Monetary Policy under DNWR

A.1 A model with staggered wage and price setting

This section outlines the central bank's problem under a more general New Keynesian model, where wages as well as prices are now Calvo rigid. Under the assumption of staggered wage setting, workers specialised in any given labour type can reset their nominal wage only with probability $1-\theta_w$, independently of the time elapsed since their last adjustment. The labour union seeks to maximise the labour type's utility subject to the sequence of labor demand schedules, outlined in the main text. In this problem the labour union must take into account the future probability of setting their wage. Given the assumed wage setting strcture, the evolution of the aggregate wage index is given by

$$W_t = \left[\theta_w W_{t-1}^{1-\epsilon_w} + (1-\theta_w)(W_t^*)^{1-\epsilon_w}\right]^{\frac{1}{1-\epsilon_w}}$$

Log-linearising around the zero wage inflation steady state yields

$$w_t = \theta_w w_{t-1} + (1-\theta_w) w_t^*$$

Combining the log-linearised wage setting rule from the labour unions problem discussed above[16] and the previous equation, and letting $\pi_t^w = w_t - w_{t-1}$ the wage inflation equation is

$$\pi_t^w = \beta E_t\{\pi_{t+1}^w\} - \lambda_w \hat{\mu}_t^w$$

where $\lambda_w \equiv \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w(1+\epsilon_w\varphi)}$. The wage markup is $\hat{\mu}_t^w = \omega_t - mrs_t - \mu^w$.[17] Therefore it can be shown that

$$\pi_t^w = \beta E_t\{\pi_{t+1}^w\} + \kappa_w \tilde{y}_t - \lambda_w \tilde{\omega}_t$$

[16] The problem is standard and therefore not derived algebraically. For the full problem outlined in detail see Galí (2015)

[17] The wage markup can also be written in terms of the unemployment gap, such that $\varphi \hat{u}_t = \hat{\mu}_t^w$, where $\hat{u}_t \equiv u_t - u^n$.

where $\{_w \quad _w(\quad + \frac{'}{1}) \; .$ By de ning the real wage gap as the di erence between the real wage and the natural real wage (real wage with no rigidities), $\tilde{!}_t \quad !_t \quad !^n_t$, hence

$$\tilde{!}_t \quad \tilde{!}_{t \; 1} + \;^w_t \quad ^p_t \quad !^n_t :$$

In the main text it is assumed that wages are exible, $_w = 0$ which therefore means that $_w ! 1$, however with Calvo wages the wage in ation equation takes the place of the real wage equation.

A.2 The optimal monetary policy problem

The central bank, under optimal policy with commitment, seeks to minimise equation (31)

$$W = \frac{1}{2}E_0 \sum_{t=0}^{1} \;^t \Big[\; + \frac{' \; +}{1 \qquad} \; \tilde{y}_t^2 + \frac{p}{p}(\;^p_t)^2 + \frac{_w(1 \qquad)}{w}(\;^w_t)^2 \Big] \tag{31}$$

subject to equation (32), (33) and, (34) for $t = 0; 1; 2; \ldots :$

$$^p_t = \; E_t f \;^p_{t+1} g + \{_p \tilde{y}_t + \;_p \tilde{!}_t \tag{32}$$

$$^w_t = \max f \; E_t f \;^w_{t+1} g + \{_w \tilde{y}_t \quad _w \tilde{!}_t \; ; \; \log(\;) g \tag{33}$$

$$\tilde{!}_t \quad \tilde{!}_{t \; 1} + \;^w_t \quad ^p_t \quad !^n_t \tag{34}$$

With the Lagrange multipliers $_{1;t}$, $_{2;t}$, $_{3;t}$, respectively. The rst order conditions below must hold when the DNWR constraint, $^w \quad \log(\;)$, is not binding:

$$+ \frac{' \; +}{1 \qquad} \; \tilde{y}_t + \{_p \;_{1;t} + \{_! \;_{2;t} = 0 \tag{35}$$

$$\frac{p}{p} \;^p_t \qquad _{1;t} + \;_{3;t} = 0 \tag{36}$$

$$\frac{{}_{w}(1 \qquad)}{{}_{w}} \; {}^{w}_{t} \qquad {}_{2;t} \qquad {}_{3;t} = 0 \tag{37}$$

$$ {}_{p} \; {}_{1;t} \qquad ! \; {}_{2;t} + \; {}_{3;t} \qquad E_{t} f \; {}_{3;t+1}\, g = 0 \tag{38}$$

and slackness conditions

$$ {}_{2;t} \qquad 0; \qquad {}^{w}_{t} \qquad \log(\ ); \qquad {}_{2;t}(\ {}^{w}_{t} \qquad \log(\ )) = 0$$

as well as initial conditions $_{1;\ 1} = \ _{2;\ 1} = 0$ and an initial condition for ⊦ $_{1}$. Whenever the DNWR constraint is not binding the model is standard and di erentiable. However, when the DNWR constraint binds equation (37) does not hold, which is communicated to the Levenberg-Marquardt mixed complementarity problem (LMMCP) solver through inserting ${}^{w}_{t}$ log( ) under the mixed complementary problem (mcp) tag for equation (37) into the Dynare mod le.[18] Thereby I avoid using the complementary slackness condition that would give rise to a singular Jacobian. This procedure is equivalent to solving for the optimal monetary policy with commitment for the non-negativity constraint of the zero lower bound in Chapter 5 of Galí (2015).

B Derivation of the theoretical model

Appendix B houses the equilibrium equations for the full non-linear New Keynesian model with Downward Nominal Wage Rigidities. In the latter part of this section the steady state of this model is outlined. Since the nominal wage is not constrained in the steady-state the lagrange multiplier associated with the downward nominal wage rigidity constraint is zero, such that $_{t} = 0$, and the steady state equations are similar to those found in most medium-scale New-Keynesian models.

[18] The LMMCP solver is used instead of Occbin by Guerrieri and Iacoviello (2015) to solve the optimal monetary policy.

B.1 Detailed derivation of Internalised DNWR

The household is the monopoly supplier of labour within this model and therefore is the wage setter. One can think that the household forms a trade union per differentiated skill j and sets wages to maximise utility whilst adhering to the demand for differentiated labour, their budget constraint and the Downward Nominal Wage Rigidity.

$$\max_{W_t(j)} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t(j)^{1+\varphi}}{1+\varphi} \right] Z_t \right\}$$

subject to:

$$N_t(j) = \left(\frac{W_t(j)}{W_t} \right)^{-\epsilon_w} N_t$$

$$(\lambda_t) \quad P_t C_t + E_t[Q_t D_{t+1}] \leq D_t + W_t(j) N_t(j) - T_t$$

$$(\mu_t) \quad W_t(j) \geq W_{t-1}(j)$$

Reformulating this problem as a Lagrange, substituting the demand for labour type j given by $N_t(j)$ and $\Pi_t^W(j) = \frac{W_t(j)}{W_{t-1}(j)}$:

$$\mathcal{L} = \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{\left(\frac{W_t(j)}{W_t} \right)^{-\epsilon_w (1+\varphi)} N_t^{1+\varphi}}{1+\varphi} \right] Z_t$$

$$-\lambda_t \left(D_t + W_t(j) \left(\frac{W_t(j)}{W_t} \right)^{-\epsilon_w} N_t - T_t - P_t C_t - E_t[Q_t D_{t+1}] \right) + \mu_t \left(\Pi_t^W(j) - 1 \right)$$

$$\frac{\partial L}{\partial W_t(j)} = \frac{{}_w(1+\ )}{W_t(1+\ )} \left(\frac{W_t(j)}{W_t} \right)^{w(1+\ )\ 1} N_t^{1+}\ Z_t + {}_t(1\quad {}_w) \left(\frac{W_t(j)}{W_t} \right)^{w} N_t$$

$$+ \quad {}_t \frac{1}{W_{t\ 1}(j)} \qquad {}_{t+1} \frac{W_{t+1}(j)}{W_t(j)^2} = 0$$

Using ${}_t = \frac{1}{P_t} C_t \quad Z_t$ from the Household's FOC of consumption and the solution of each labour type is identical, therefore $W_t(j) = W_t$)

$$\frac{w}{W_t} N_t^{1+}\ Z_t + \frac{1}{P_t} C_t \quad Z_t(1\quad {}_w) N_t + {}_t \frac{1}{W_{t\ 1}} \qquad {}_{t+1} \frac{W_{t+1}}{W_t^2} = 0$$

$$_w N_t^{1+}\ Z_t + \frac{W_t}{P_t} C_t \quad Z_t(1\quad {}_w) N_t + {}_t \frac{W_t}{W_{t\ 1}} \qquad {}_{t+1} \frac{W_{t+1}}{W_t} = 0$$

$$_w N_t^{1+}\ Z_t + \frac{W_t}{P_t} C_t \quad Z_t(1\quad {}_w) N_t + {}_t\ {}^w_t \qquad {}_{t+1}\ {}^w_{t+1} = 0$$

Hence, we arrive at the solution presented in the main body of the paper. When the DNWR constraint does not being ${}_t = 0$ and the wage is set exibly as a markup over the marginal rate of substitution.

$${}^w_t \quad {}_t = (\ {}_w \quad 1) \frac{W_t}{P_t} Z_t C_t \quad N_t \quad {}_w N_t^{1+}\ Z_t + \quad {}_{t+1}\ {}^w_{t+1}$$

Additionally:

Complementary slackness:

$${}_t(\ {}^w_t \qquad) = 0$$

Feasibility (non-negativity constraints):

$${}^w_t \qquad 0$$

$${}_t \quad 0$$

must hold.

B.2 Equations of the full model

The equations of the model outlined in Section 6 where the labour union internalises the downward nominal wage rigidity constraint is outlined below:

$$Q_t = \frac{\frac{C_{t+1}}{C_t}^{(\quad)} \frac{Z_{t+1}}{Z_t}}{{}_{t+1}} \tag{39}$$

$$R^n{}_t = \frac{1}{Q_t} \tag{40}$$

$$Y_t = A_t \quad \frac{N_t}{S_t}^{1} \tag{41}$$

$$R^n{}_t = \quad {}_{t+1}\, R^r{}_t \tag{42}$$

$$R^n{}_t = \frac{ss}{} \quad \frac{t}{ss} \quad \frac{Y_t}{(Y)}^{y} \quad e^{t} \tag{43}$$

$$C_t = Y_t \tag{44}$$

$$\log(A_t) = \ {}_a \log(A_{t\ 1}) + \text{"}_{at} \tag{45}$$

$$\log(Z_t) = \ {}_z \log(Z_{t\ 1}) \quad \text{"}_{zt} \tag{46}$$

$$MC_t = \frac{\frac{W}{P}_t}{S_t \frac{Y_t (1 \quad)}{N_t}} \tag{47}$$

$$1 = \quad {}_t{}^{1} + (1 \quad) \quad {}_t{}^{1} \tag{48}$$

$$S_t = (1 \quad) \quad {}_t{}^{\frac{(\ )}{1}} + \quad {}_t{}^{\frac{}{1}}\ S_{t\ 1} \tag{49}$$

$$t^{1+} \overline{1} = \frac{\frac{x_{1t}}{x_{2t}}}{1} \tag{50}$$

$$x_{1t} = MC_t Y_t Z_t C_t^{(\ )} + \quad {}_{t+1} \ {}^{+} \overline{1} \ x_{1t+1} \tag{51}$$

$$x_{2t} = Y_t Z_t C_t^{(\ )} + \quad {}_{t+1} \ {}^{1} x_{2t+1} \tag{52}$$

$${}^{w}_{t} \quad {}_{t} = (\ {}_{w} \quad 1) \frac{W_t}{P_t} C_t \quad N_t \quad {}_{w} N_t^{1+} \quad + \quad {}_{t+1} \ {}^{w}_{t+1} \tag{53}$$

$$\frac{W_t}{P_t} = \frac{{}^{w}_{t}}{{}_{t}} \frac{W_{t \ 1}}{P_{t \ 1}} \tag{54}$$

$${}_{t} = \ {}_{a} \ {}_{t \ 1} + \ {}_{t} \tag{55}$$

$$V_t = Z_t \quad \log(C_t) \quad \frac{N_t^{1+'}}{1+'} \quad + \quad V_{t+1} \tag{56}$$

$${}_{t}(\ {}^{w}_{t} \quad) = 0 \tag{57}$$

$${}^{w}_{t} \qquad 0 \tag{58}$$

$${}_{t} \quad 0 \tag{59}$$

B.3 Steady State

State state of the model with internalised DNWR is outlined below. The steady state is not a ected by the DNWR constraint.

$$A = 1$$

$$= 0$$

$$Z = 1$$

$$= \frac{h_1 \qquad {}_1 i}{1} \frac{1}{1}$$

$$S = \frac{(1 \qquad) \quad \overline{1}}{1 \qquad \overline{1}}$$

$$MC = \frac{1}{\quad} \frac{1+}{1} 1 \qquad + \frac{}{1} \frac{1}{1 \qquad 1}$$

$$Q = \frac{}{}$$

$$R = \frac{1}{Q}$$

$$r = \frac{R}{}$$

$$N = {}^{h} MC(1 \qquad) \frac{w \quad 1}{w} S \qquad + \quad {}^{i} \frac{1}{(1 \quad) + ' +}$$

$$C = A \frac{N}{S}^{1}$$

$$Y = C$$

$$\frac{W}{P} = w = \frac{MC \quad S \quad Y(1 \qquad)}{N}$$

$$x_1 = \frac{C \quad Y \quad MC}{1 \qquad \frac{+}{1}}$$

$$x_2 = \frac{C \quad Y}{1 \qquad\qquad {}_1}$$

$$= 0$$

C Computational technique

Two computation techniques have been used in this project. Firstly, Occbin by Guerrieri and Iacoviello (2015) is used as a rst attempt to analyse the e ect of DNWR on a standard New-Keynesian Model. Latterly, Smolyak Projection Method by Smolyak (1963) is used to provide more realistic analysis as it allows the agents to understand that the DNWR constraint exists. Below I outline both of these techniques used.

C.1 Occbin

Most of the model simulations, impulse response functions and welfare losses were calculated using Dynare[19], an extension to Matlab used for DSGE models. Dynare cannot typically be used when there is an occasionally binding constraint such as the DNWR, however, with help of the Occbin toolbox seen in Guerrieri and Iacoviello (2015) it is possible. Occbin uses rst order pertubation but allows the solution to be highly non-linear. One disadvantage is that all agents within the model have no prior knowledge of the existence of the occasionally binding constraint, and therefore this technique does not capture precautionary behaviour.

At the start of the period the model is at the steady state and then when the households' wish to lower the nominal wage after the monetary policy shock, the model switches to that of the binding constraint and the wage reduction is forced to be sluggish. Appendix C.2 highlights a projection method, which provides a global solution, used to interalise this occasionally binding constraint and will form the

C.2 Smolyak Approximation

Section 6 displays the non-linear model with positive trend in ation and an occasionally binding constraint that the households maximise over. The model is solved using the Smolyak collocation method laid out in Malin et al. (2011) and implemented for a New-Keynesian model with a Zero-Lower-Bound constraint in

[19]The dynare les used were adapted from those created by Dr Johannes Pfeifer to replicate Galí (2015) and provided freely for use, as of which I am extremely grateful.

Fernández-Villaverde et al. (2015). My solution technique closely follows the exercise provided by Fernández-Villaverde et al. (2015). Smolyak collocation allows for more state variables than other common projection methods as the number of terms of the approximating polynomial and grid points do not grow exponentially and therefore do not suffer as much as other techniques from the curse of dimensionality. One prominent example is Fernández-Villaverde and Levintal (2016), which uses 12 state variables and still retains accuracy and speed of computation.

Smolyak's algorithm introduced in Smolyak (1963) is a numerical technique using a sparse grid to efficiently solve multi-dimensional hypercubes. The technique ordered and selected the solution to a tensor-product rule importance of finding the quality of approximation to the problem. Smolyak's algorithm was then adapted by Krueger and Kubler (2004) to be used in an economic setting.

Following the steps found in the technical appendix of Fernández-Villaverde et al. (2015) I start by defining a state vector:

$$\mathsf{S}_t = (S_{t\ \ 1}; A_t; Z_t; w_{t\ \ 1})$$

With the exogenous states in logs:

$$\hat{\mathsf{S}}_t = (S_{t\ \ 1}; \log(A_t); \log(Z_t); w_{t\ \ 1})$$

The equilibrium functions $f = (f^1; f^2; f^3; f^4)$ characterize the dynamics of the model:

$$\log(C_t) = f^1(\hat{\mathsf{S}}_t)$$

$$\log(\ \ _t) = f^2(\hat{\mathsf{S}}_t)$$

$$\log(x_{1t}) = f^3(\hat{\mathsf{S}}_t)$$

$$\log(\ \ _t^w) = f^4(\hat{\mathsf{S}}_t)$$

$$_t = f^5(\hat{\mathsf{S}}_t)$$

To define the hypercube (grid points) we then choose bounds on the state variables around their steady state levels. The bounds for the exogenous state variables are determined by their unconditional standard deviation.

Then to solve for f I use a time-iteration procedure:

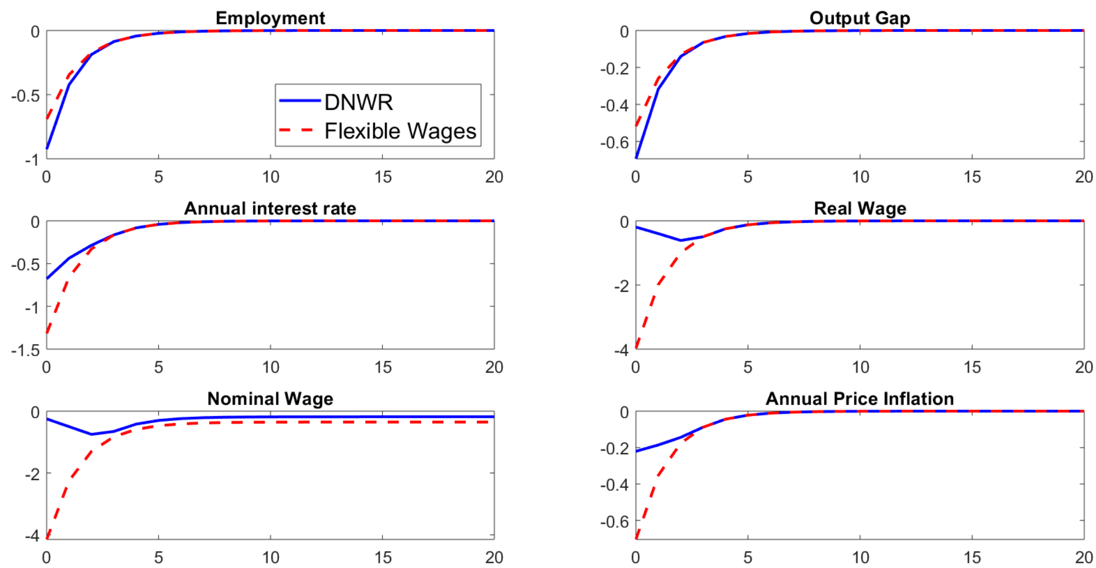
- Guess on : $\{ \ _t; \ ^{w}_{t}\}$
- Update state to obtain: $= \{S_t; \log(A_{t+1}); \log(Z_{t+1}); w_t\}$
- Using the state today and weights from a monomial rule calculate expectations of time $t + 1$ variables in the model.
- Check whether initial guess was correct by using the euler equation, real wage equation and complementary slackness for the occasionally binding constraint - iterate over guess if not correct.
- With the time t equilibrium found at each of the collocation points, check if they differ from the $t + 1$ values. If they are similar up to a tolerance level then stop.

D Additional figures or tables

D.1 Negative Demand Shock- amplification

Figure 8 outlines the response to a negative demand shock when the economy suffers from DNWR in comparison to the economy under flexible wages. Due to the fall in demand, wages fall, however as wages hit the DNWR constraint they are artificially higher. These high wages are a cost to the firm and therefore employment falls more relative to when wages are flexible. In this labour-driven economy the fall in the output gap is amplified when wages are downwardly rigid compared to their flexible counterpart.

Figure 8: Negative demand shock with DNWR

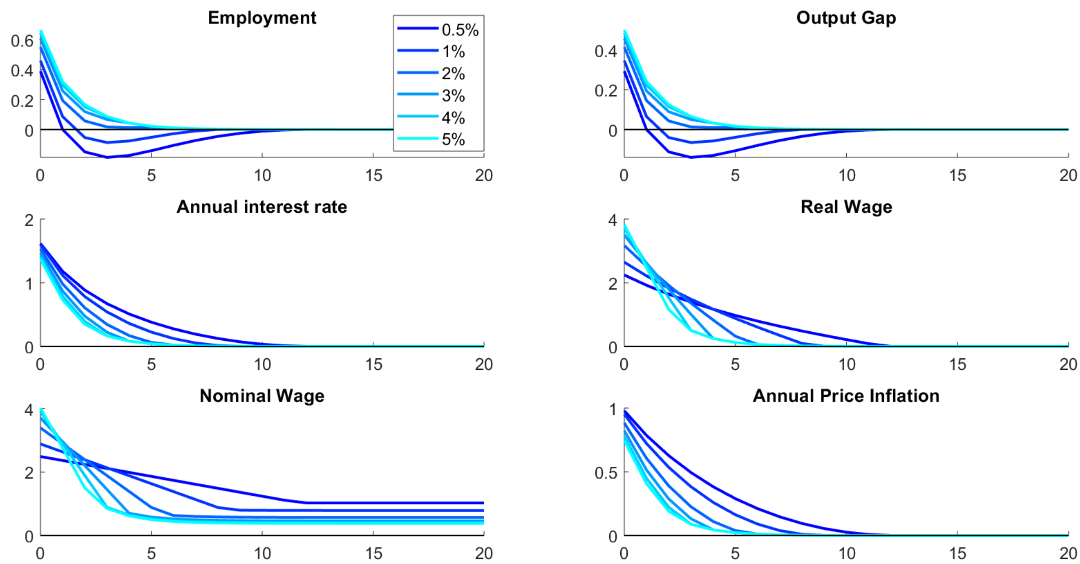


Impulse response for variables facing a negative demand shock. The demand shock follows an AR(1) with $_z = 0{:}5$ and $= 1$. The Taylor rule is $i_t = 0{:}01 + 1{:}5 \; ^p_t + 0{:}125\hat{y}_t + \; _t$.

D.2 Varying the degree of DNWR

As outlined in Figure 9, the degree to which wages are allowed to fall is crucial in driving the boom-bust cycle from a positive demand shock, the amplification from a negative demand shock and also the motivation for a higher trend inflation in the extended model. In the figure below, the degree of DNWR is varied such that wages are allowed to fall from 0.5% a year to 5%. Currently the baseline model allows wages to fall by 1% a year, which corresponds to $= 0.9975$, a figure within the bounds of the estimated DNWR for various countries by Schmitt-Grohé and Uribe (2016). In contrast wages are allowed to fall by 4% per year , $= 0.99$, in the baseline case of Schmitt-Grohé and Uribe (2016). As can be seen in the figure below, allowing for nominal wages to fall 2% per year removes the boom-bust result highlighted in the main text of the paper. However, in this case the outcome of the economy is still distorted as the downward nominal wage rigidity constraint is still binding. The binding of the constraint, even if the boom-bust cycle is not apparent, still causes a sharper fall in employment and the output gap compared to a case where the wage is allowed to fall by more (e.g. 5% per year).

Figure 9: Varying degrees of Downward Nominal Wage Rigidity

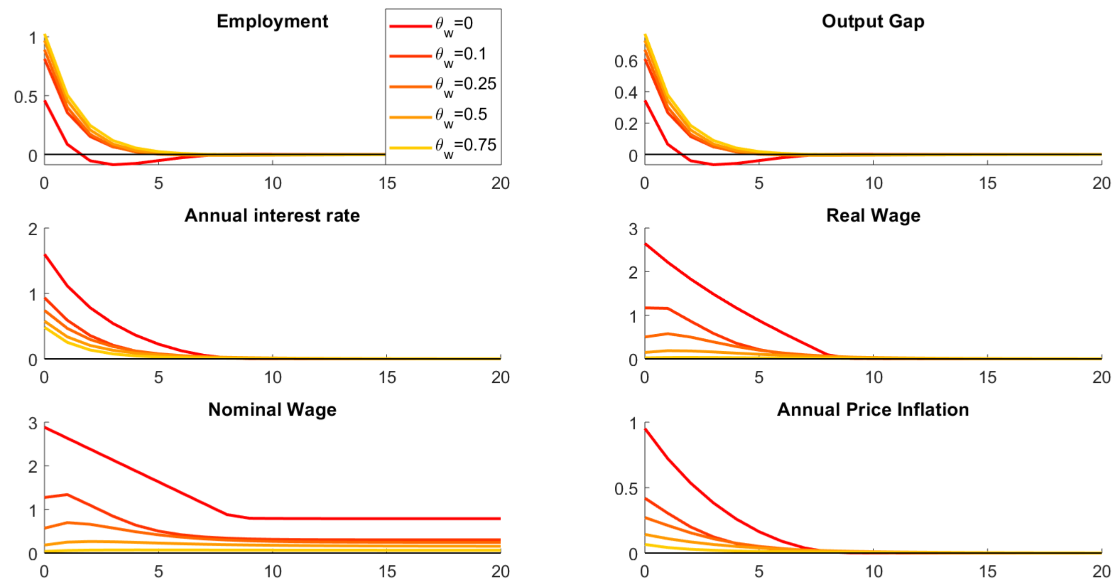


Impulse response for variables facing a positive demand shock. The demand shock follows an AR(1) with ${}_z = 0.5$ and $= 1$. The degree to which wages are allowed o fall per year are varied from 0.5% to 5%. The Taylor rule is $i_t = 0.01 + 1.5\,{}^{p}_{t} + 0.125\hat{y}_t + {}_t$.

D.3 Varying the degree of wage stickiness

The previous models abstracted away from wage stickiness, the phenomenon that wages may be nominally rigid as wage changes are often infrequent. I follow the standard procedure from adding Calvo wage rigidity outlined in Galí (2015) to the New Keynesian model with DNWR and vary the degree to which these wages are rigid. In the benchmark case, $_w = 0.75$, and as nominal wages are muted following a positive demand shock outlined in Figure 10 the DNWR does not impact the economy. In fact, it is only when wages are fully flexible upwards do we see the boom-bust cycle returning to the economy due to a positive demand shock. Adding in an extra rigidity, Calvo wages, to any degree that resembles the parameter values commonly used in the literature removes the boom-bust result. Furthermore, even when $= 0.1$, and hence wages are allowed to adjust regularly but not fully flexibly, we see the downward nominal wage rigidity constraint binding only for a few periods and the outcome to the output gap is very similar to when $= 0.75$. Therefore, the introduction if Calvo wage rigidity into the model, somewhat dominates the downward nominal wage rigidity.

Figure 10: Varying degrees of Calvo Wage Rigidity



Impulse response for variables facing a positive demand shock. The demand shock follows an AR(1) with $_z = 0{:}5$ and $= 1$. The degree of wage rigidity is varied, $_w$ goes from 0, full wage flexibility, to $_w = 0{:}75$, the benchmark case in the New Keynesian model with wage rigidity. The Taylor rule is $i_t = 0{:}01 + 1{:}5 \; {}^p_t + 0{:}125\hat{y}_t + \; _t$.

The culmination of Figure 9 and 10 highlight the important prerequisites needed to drive the results laid out in the main body of the paper. Under the current calibration and adapting the simple New-Keynesian model flexible wages and a high enough degree of downward nominl wage rigidity is needed for boom-bust cycles to be present following a positive demand shock. However, even with relatively accommodate wage deflation asymmetric response of the economy to symmetric shocks would still persist. Furthermore, Figure 10, when wages are sufficiently sticky, closely resembles the response of the economy in the extended model where households internalise the downward nominal wage rigidity constraint and maximise their utility and wage setting decision taking it into account, as seen in Figure 7.

D.4 Welfare loss for negative shock

With an occasionally binding constraint the response of a central bank following an interest rate rule or an optimal monetary policy can be asymmetric. Therefore it is important to look at welfare loss for different interest rate rules under positive and negative shocks separately. Hence, table 5 displays the welfare loss from negative shocks to provide a comparison with table 2 found in the main body of the paper.

As with a positive demand shock the optimal policy is able to change the interest rate such that no welfare is lost from the shock. The optimal simple rule in this scenario also does well, for negative technology and demand shocks. Strict price targeting performs well under demand shocks however this regime performs poorly under technology shocks relative to the optimal monetary policy or optimal simple rule.

Table 5: Evaluation of MP rules following **negative** Technology and Demand Shocks

	Optimal	Strict Targeting		Simple Rules 1, 2, & 3		
		Price	Wage	Taylor	Galí	OSR
Technology shocks						
$(\ ^{p})$	0.004	0	0.0134	0.063	0.031	0.008
$(\ ^{w})$	0.105	0.047	0	0.089	0.036	0.042
$(\hat{y})$	0.021	0.254	0.020	0.040	0.020	0.008
L	0.005	0.840	0.133	1.469	0.364	0.024
Demand Shocks						
$(\ ^{p})$	0	0	0	0.014	0.015	0.010
$(\ ^{w})$	0	0	0	0.096	0.099	0.093
$(\hat{y})$	0	0	0	0.138	0.083	0.029
L	0	0	0	0.120	0.075	0.026

Simple Rule 1 (Taylor) is the Taylor rule with $i_t = 0.01 + 1.5\ {}^{p}_{t} + 0.125\hat{y}_t + \ _t$ and Simple Rule 2 (Galí) is $i_t = 0.01 + 1.5\ {}^{p}_{t} - 0.5\hat{u}_t$. Simple Rule 3 (OSR) is $i_t = 0.01 + 4.0\ {}^{p}_{t} - 2.5\hat{u}_t$.

D.5 Perfect Foresight Solution

Figure 11 highlights the difference between the model with flexible wages, when wages are downwardly rigid and when wages are downwardly rigid but the labour union internalises the downward nominal wage rigidity constraint as explained further in Section 6. Once the labour union internalises the DNWR constraint the